Grade 12

Tutorials

2008

GRADE 12 TUTORIALS

1. Given the sequence 2; 5; 8; 11; …

   1.1 Determine the 250th term.

   1.2 Which term is 302?

   1.3 How many terms must be added to obtain a sum of 610?

2. A geometric sequence with positive terms has a 4th term of 250 and a 6th term of 6 250. Find the first term and the common ratio.

3. The first four terms of a sequence are $81,\ x,\ y$ and 3.

   3.1 Determine the values of x and y if the sequence is:

   3.1.1 arithmetic

   3.1.2 geometric.

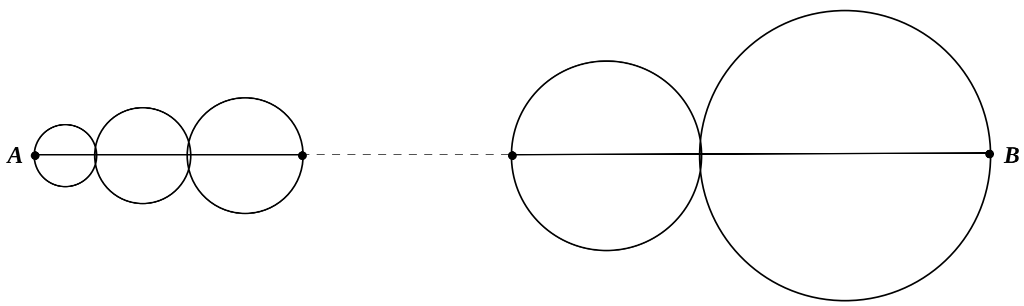
   3.2 The geometric series $81+x+y+3+...$ is convergent. Determine S_∞.

4. Calculate the sum $6+1-4-9...-239$.

5. Which term of the sequence $\frac{1}{6};\frac{1}{3};\frac{2}{3};...$ is $\frac{256}{3}$?

6. Evaluate $\sum_{k=1}^{40} 5.2^{k-1}$.

7. A number of circles touch each other as shown below.



The area of the smallest circle is $4\pi\ cm^2$ and each consecutive circle has an area $\frac{9}{4}$ times that of the previous one. If the distance $AB = \frac{665}{8}cm$, how many circles are there?

8. A rubber ball dropped from a height of 15 m loses 20% of its previous height at each rebound.

Calculate:

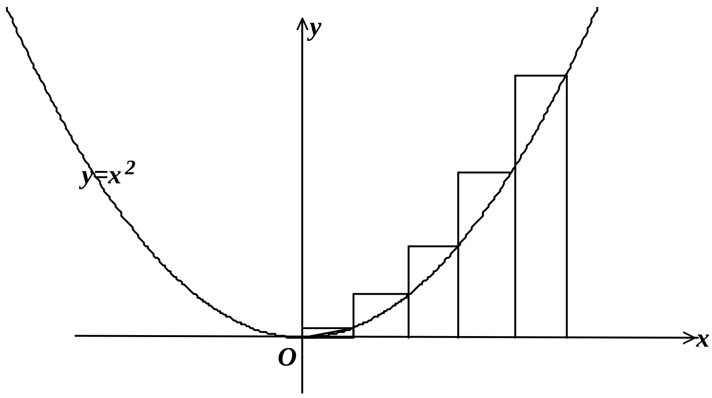
8.1 the height to which the ball will rise on the second rebound;

8.2 the number of times it will rise to a height of more than 3 m;

8.3 the total distance the ball will travel before it comes to rest.

9. Christina gets a salary of R4 000 a month and is to receive an increase of R500 per month each year. Lindiwe is getting only R2 500 a month, but she will receive an increase of R750 a month each year. After how many years will Lindiwe's salary exceed Christina's?

10. The area under the curve $y = x^2$ between $x = 0$ and $x = 1$ is being approximated by adding the areas of the five rectangles sketched below. Each has a width of 0,2 units.



10.1 Calculate the sum of the areas of the five rectangles.

Suppose that the same area is to be calculated by using n rectangles instead of five.

10.2 Write down the area of the n rectangles in sigma notation.

10.3 If it is given that $\sum_{i=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$, calculate the area of 100 rectangles, drawn as above for $x \in [0;1]$

10.4 Which answer do you think is the more accurate approximation for the area between the curve $y = x^2$ and the y axis for $x \in [0;1]$: 10.1 or 10.3? Explain.

Question 1

The relation $f(x) = -2x + 3$ is a function.

1.1 Explain in words what is meant by the term "function".

1.2 Demonstrate algebraically that the domain element -1 maps onto the range element 5.

1.3 Draw a sketch graph of f .

1.4 Explain how you can determine from the graph whether or not f is a function.

1.5 Explain how you can determine from the graph that f is a one-to-one function.

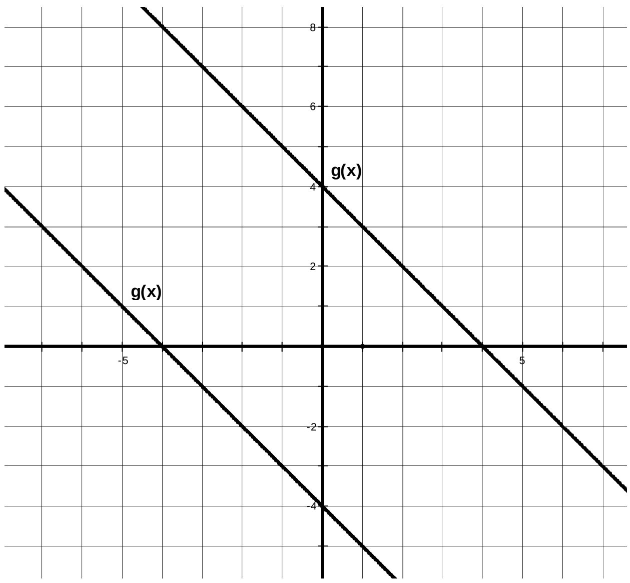
Question 2

If $g(x) = \pm\sqrt{16} - x$

2.1 Demonstrate, using a numerically example, that g is not a function.

2.2 Explain in words why $g(x)$ is not a function.

2.3 The diagram below represents the sketch graph of g . If the domain of g were restricted to $x > 0$, would g be a function? Explain your answer.

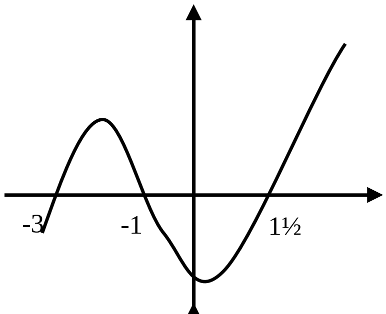


Question 3

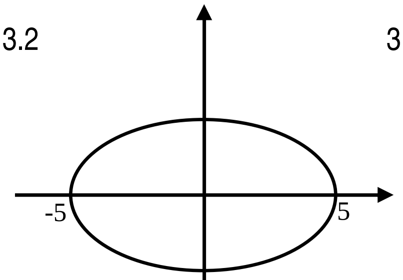
Each of the following diagrams represents a sketch graph of a relation.

- State whether or not the relation is a function.
- If the relation is a function, state whether it has a one-to-one mapping or a many-to-one mapping.
- If the relation is not a function, explain how the range should be restricted in order for it to be a function.

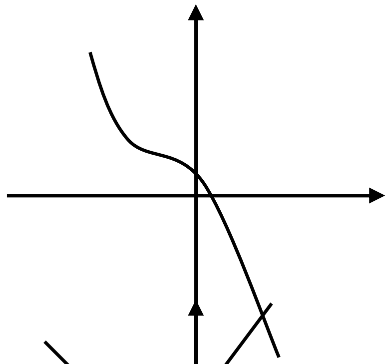
3.1



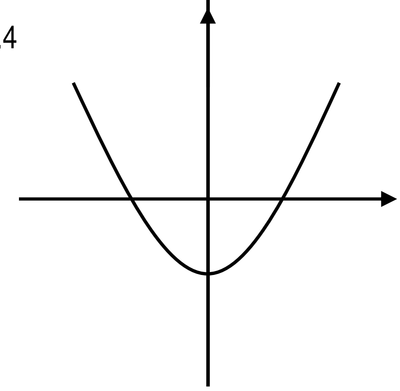
3.2



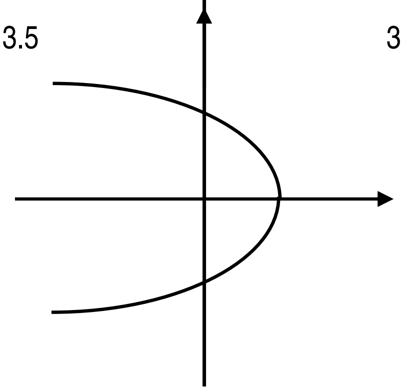
3.3



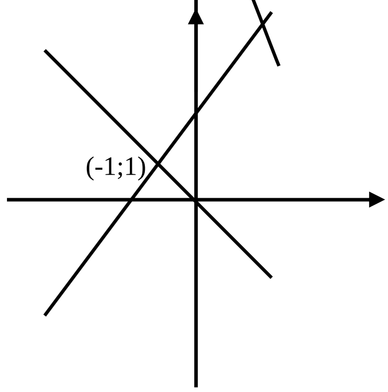
3.4



3.5



3.6



Question 4

State, giving an explanation, which of the following functions has a one-to-one mapping:

4.1 $f(x) = 5$

4.2 $g(x) = 5^x$

4.3 $h(x) = \frac{5}{x}$

4.4 $j(x) = \sin 5x$

Question 5

5.1 Draw a sketch graph of $f(x) = x^2$.

5.2 Using a reflection, sketch the graph of $f^{-1}(x)$.

5.3 Find the equation of $f^{-1}(x)$.

5.4 Explain why $f^{-1}(x)$ is not a function.

5.5 Explain how you would restrict the domain of $f(x)$ so that $f^{-1}(x)$ is a function.

Question 6

As a rule of thumb, 8 kilometres equals 5 miles. In order to convert miles to kilometres, the mathematical function $f(x)=\frac{8}{5}x$ may be used, where *x* is the number of miles and *f(x)* is the equivalent number of kilometres.

6.1 Is there a one-to-one relationship between miles and kilometres? Explain your answer.

6.2 On graph paper, draw the graph of *f* for the domain $0 \leq x \leq 30$.

6.3 Use the graph to read off the kilometre equivalent of 15 miles.

6.4 Determine the equation of f^{-1}, the inverse of *f*.

6.5 Explain, in practical terms, what f^{-1} represents. (When would you use f^{-1}?)

6.6 Draw the graph of f^{-1} on the same grid.

6.7 Demonstrate by taking several readings from your graphs that *f* and f^{-1} are symmetrical to each other with respect to the line $y=x$.

Question 7

If $f(x)=5^x$, which of the sketch graphs ((a) – (h) below) represents:

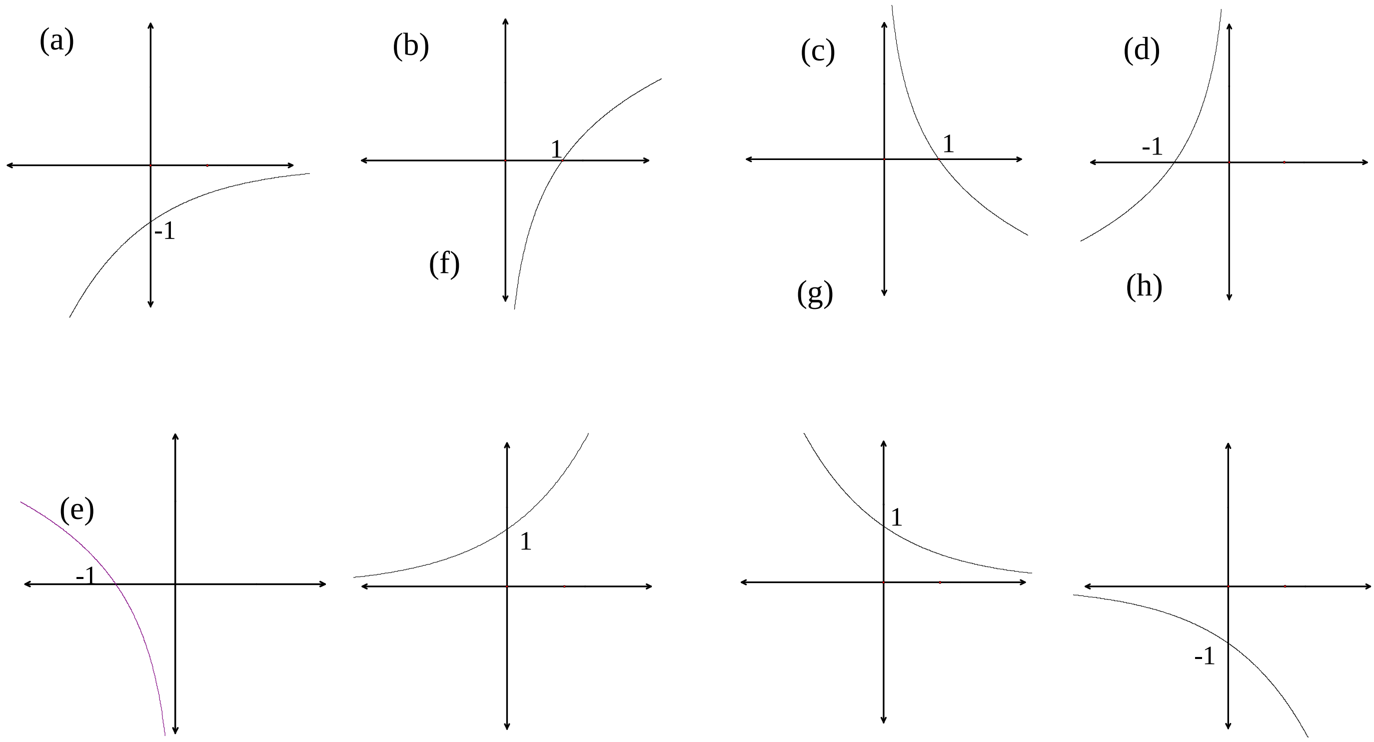
7.1 $f(x)$

7.2 $g(x)$, which is the reflection of *f* in the line $x=0$

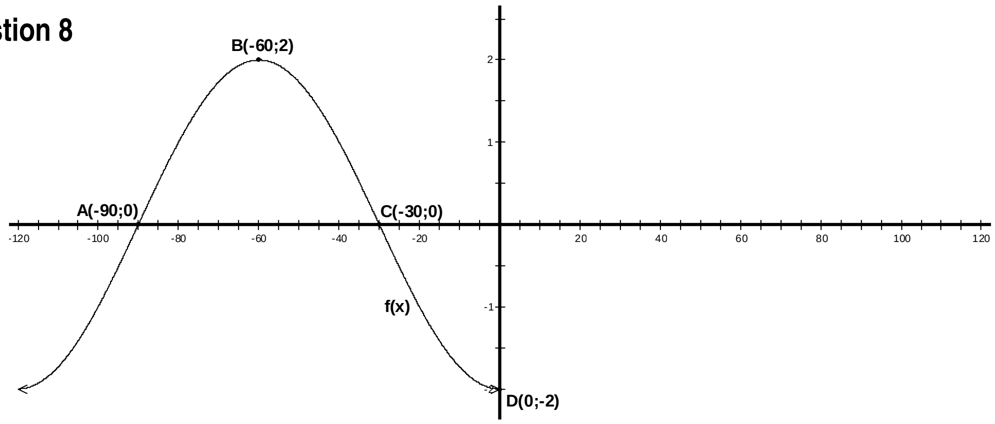
7.3 $g^{-1}(x)$, which is the reflection of *g* in the line $y=x$

7.4 $h(x)$, which is the reflection of $g^{-1}(x)$ in the line $y=0$

7.5 Give the equations of each of the graphs (a) – (h)



Question 8



The diagram above represents the graph of $f(x) = a\cos 3x$ for the domain $-120° \leq x \leq 0°$.

8.1 Determine the value of *a*.

8.2 Using the periodicity of the function, extend the graph over the domain $-120° \leq x \leq 120°$. Give the co-ordinates of the intercepts and turning points.

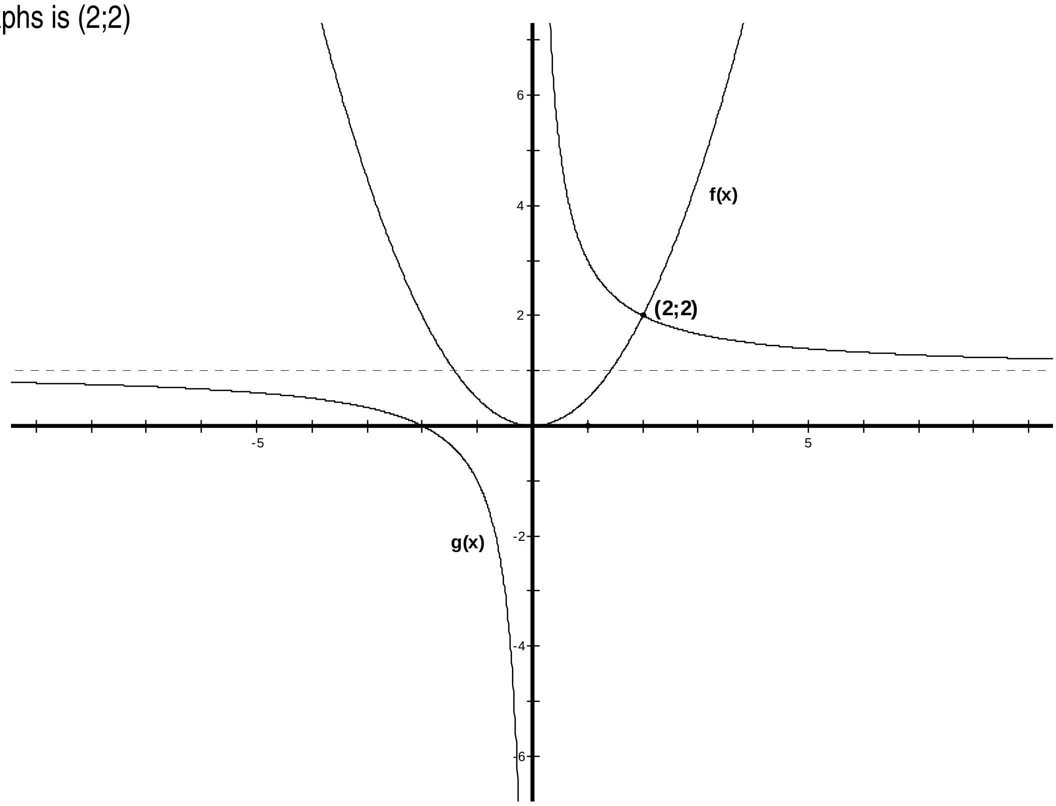
8.3 Using your knowledge of the trigonometric function $f(x) = a\cos 3x$, show that the curve is symmetrical about the *y*-axis.

8.4 Write down the new equation of *f* if it were shifted horizontally $15°$ to the left.

8.5 Explain the influence that the following transformation would have on the graph: $(x; y) \rightarrow (x; y-1)$, as well as the change in the range of the function as a result of the transformation.

Question 9

The graphs of $f(x)=ax^2$ and $g(x)=\frac{k}{x}+1$ are shown in the diagram below. The point of intersection of the two graphs is (2;2)



9.1 Determine the values of *a* and *k*.

9.2 Give the co-ordinates of the turning point of *f* and the equation of the axis of symmetry.

9.3 Using the point (2:2) and symmetry, give the co-ordinates of one other point on *f*. Explain which symmetry you used and how you arrived at your answer.

9.4 Give the equation(s) of the asymptotes of *g*.

9.5 Give one value of *x* for which $f(x)+g(x)=2$

9.6 For which values of *x* is $f(x)$ increasing?

9.7 On the diagram, sketch the graph of $h(x)=x$ and determine the point(s) of intersection of *g* and *h*.

9.8 Is *h* symmetrical about the line $h(x)=x$? Substantiate your answer through calculation.

9.9 If the domain of *f* is restricted to $x\leq 0$, draw the graph of f^{-1} on the diagram.

9.10 Using symmetry between a function and its inverse, give the co-ordinates of one point on f^{-1}.

9.11 Give the range of f^{-1}.

9.12 If a negative vertical shift of -2 is applied to *f*, how will this move the graph?

Question 1

Solve for *x*:

1.1 $x^2 + 6x = 0$

1.2 $(x - 4)(x - 3) = 2$

1.3 $3x^2 + 2x - 6 = 0$

1.4 $x^2 - 3x + 2 < 0$

1.5 $\log_{\frac{1}{2}} x = -3$

1.6 $700(1{,}12)^x = 1250$

1.7 $\frac{x}{2} + \frac{12}{x} = -5$

1.8 $2^{2x-3} = 8^{x+1}$

1.9 $\frac{x+3}{x-1} > 2$

1.10 $\log_{\frac{1}{3}} x < 2$

1.11 $x(x-3) = 4(x+2)$

1.12 $(x-3)^2 < 4x$

1.13 $\log_2(-16) = x$

1.14 $(x-1)(2x^2 + 5x + 3) = 0$

1.15 $2500\left(1 + \frac{8}{100}\right)^x = 3375$

1.16 $2^x \,.\, 3^{x+2} = 120$

1.1.7 $x = \sqrt{50} - \sqrt{98}$

Question 2

Given $x^2 = 8x$

2.1 Solve for *x*.

2.2 Hence solve for k if $(k^2 - 1)^2 = 8(k^2 - 1)$

Question 3

If (*x* – 2)(*y* – 3) = 0, what can you say about *y* if

3.1 *x* = 2

3.2 *x* = 5

Question 4

For which values of x will the expression $\sqrt{\dfrac{3-x}{x+7}}$ be a real number?

Question 5

Determine the values of x for which M is a real number if $M = \sqrt{\dfrac{2}{2x+5}} + \dfrac{1}{2x}$

Question 6

What is the units digit of the following product: $2^{1999} \times 3^{2000}$?

Question 7

Solve simultaneously for x and y in each of the following systems of equations:

7.1 $y + 2x = 1$ and $y^2 + x^2 = 2$

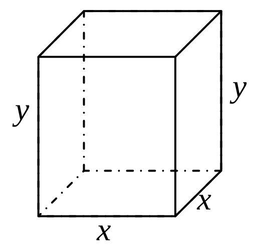
7.2 $x - 2y = 1$ and $x^2 - 2xy + y^2 - 9 = 0$

7.3 $5^{x+y} = 125$ and $2x^2 + 2y^2 = 5xy$

7.4 $\log_2(2x - y) = 1$ and $y = (x-1)(x-2)$

Question 8

A closed box has the shape of a rectangular prism with a square base. The sides of the base are x cm long. The height is y cm. The surface area of the box is 288 cm^2. The lengths of the edges are such that $2x + y = 21$.



8.1 Show that $x^2 + 2xy - 144 = 0$.

8.2 Hence, calculate the values of x and y.

Question 9

Thabo employs a certain number of workers and pays each worker the same wage. His current daily wage bill is R5 880. A labour dispute has resulted in his workers demanding a wage increase of R10 per day. Thabo claims that he cannot afford this. He claims that only if he retrenches 4 workers will he be able to give them the increase that they demand. His daily wage-bill would then be R5 850.

Question 10

Factorise each of the following:

10.1 $x^3 - 4x^2 - 11x + 30$

10.2 $-x^3 - 3x^2 - 3x + 7$

10.3 $x^3 + 3x^2 + 3x + 2$

10.4 $x^3 - 5x^2 + 7x - 3$

10.5 $2x^3 - 5x^2 - 4x + 3$

1. How long (correct to the nearest month) will it take a motor car bought for R200 000 to depreciate to R50 000 if depreciation is calculated at 16% p.a.

    1.1 on the straight line method;

    1.2 on the reducing balance?

2. Goods worth R1 500 are bought on hire purchase and payments are made over five years. If the total amount repaid is R2 400, what is the (simple) interest rate?

3. To save for an overseas trip, I deposit R12 000 now, R12 000 in a years time and a further R12 000 in two years' time. The trip, at the end of three years costs R50 000. If interest of 11,2% compounded monthly is paid for the first two years and if this rate rises to 11,5% for the rest of the time, what is the balance that I will have to pay in three years time?

4. A company buys equipment for 1,8 million rand.

    4.1 Calculate the book value of the front-end loader at the end of eight years if depreciation is calculated at 13,5% p.a. on the reducing balance.

    4.2 Calculate the expected cost of the front-end loader if inflation is estimated to be 5.5% p.a.

5. Calculate the effective interest rate if the nominal rate is:

    5.1 12% p.a., calculate monthly;

    5.2 10,5% p.a., calculated on a daily balance;

    5.3 13% p.a. calculated quarterly.

6. Calculate the nominal interest if the effective interest is:

    6.1 8,77% p.a. and interest is calculate quarterly;

    6.2 12% p.a. and interest is calculate monthly;

    6.3 10,5% and interest is calculated daily.

7. Machinery has to be replaced by a company in 10 years time. It is anticipated that the replacement cost will be R180 000.What amount should be deposited each month, into an account paying 12% p.a., compounded monthly, if the first deposit is made at the end of the first month of the first year and the last at the end of the last month in the 10^{th} year?

8. A sinking fund is established to raise R250 000 in 4 years time. What monthly payment, starting now and finishing one month before the money is required, must be paid into an account paying 10% p.a. compounded monthly?

9. Peter takes out a loan of R510 000 to be able to buy a house. He pays back the loan over a period of 20 years, starting one month after he bought the house. Interest is charged at 14%p.a., compounded monthly.

   9.1 Determine his monthly payments (correct to the nearest cent).

   9.2 How long would it take to repay the loan if he were to pay R10 000 each month?

   9.3 Did you find the answer to 9.2 surprising? Why or why not?

Question 1

Determine the following limits:

1.1 $\lim_{x\to -2}\frac{x^2-4}{x+2}$

1.2 $\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$

Question 2

Given $f(x)=-\frac{2}{x}$, and $g(x)=2-x^2$ calculate from first principles.

2.1 $f'(x)$

2.2 $g'(-1)$

2.3 $D_x[6]$

Question 3

Use the rules for differentiation to find the following derivatives:

3.1 $\frac{dy}{dx}$ if $y=\frac{5x^2-3x}{\sqrt{x}}$

3.2 $D_x[(x^2-1)(x+2)]$

3.3 $f'(x)$ if $f(x)=\frac{x^4}{4}-\frac{2x^3}{3}+x^2-x+2$

3.4 $\frac{d}{dx}\left(\frac{x^3}{3}-\frac{3}{x^3}\right)$

3.5 $\frac{dy}{dx}$ if $y=(x^2+1)(2x-3)$

3.6 $g'(x)$ if $g(x)=2\sqrt{x^3}-3\sqrt{x}+\frac{2}{\sqrt{x}}$

Question 4

$s=5t^2$ is a formula for the distance, *s,* in metres, fallen by a stone after *t* seconds if dropped from the top of a cliff .

4.1 How far has the stone fallen after 2 seconds?

4.2 What is the average speed of the stone in the third second (between *t*=2 and *t* =3)?

4.3 What will be the instantaneous speed of the stone after 3 seconds?

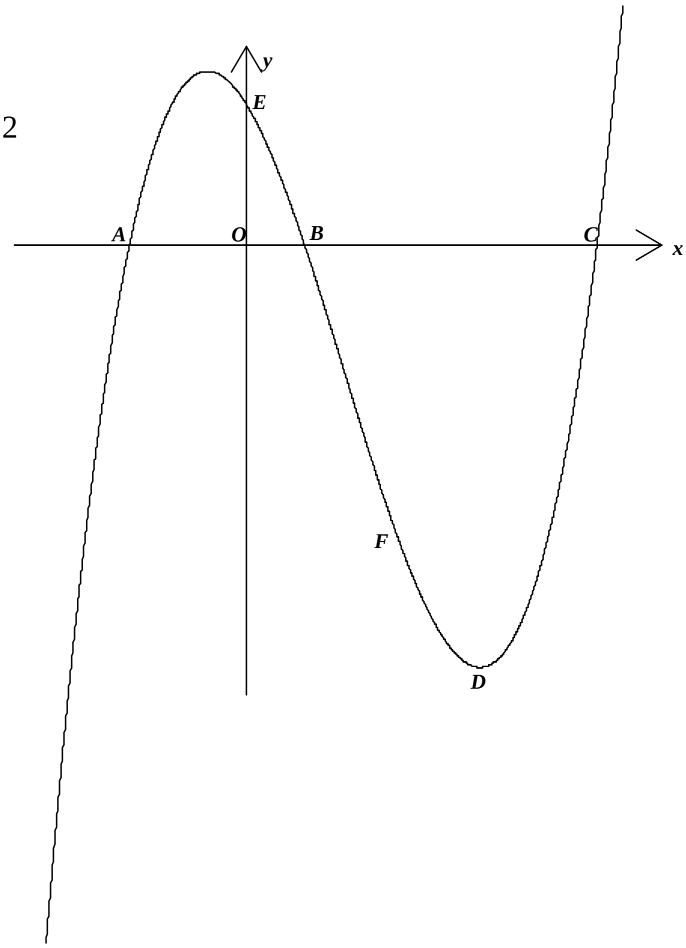
4.4 When will the instantaneous speed be 20 metres per second?

4.5 How long will it take for the stone to hit the ground if the cliff is 320 metres high?

4.6 What will be the speed of the stone when it hits the ground?

Question 5

Sketched below is $y = f(x) = x^3 - 5x^2 - 8x + 12$



Determine:

5.1 the coordinates of A, B and C

5.2 the coordinates of the local minimum D.

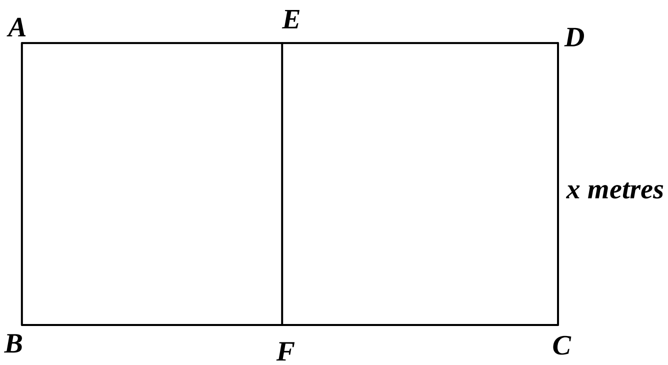
5.3 the co-ordinates of the point of inflection on the curve between B and D

5.4 the equation of the tangent to the curve at the point E where $y = f(x)$ cuts the y axis

5.5 the *x*-coordinate of the point F at which the tangent of the curve is parallel to the tangent at E.

Question 6

The area of .rectangle ABCD sketched below is 2 400 m^2. DC = x *metres*.



6.1 Express AD in terms of x.

6.2 If the rectangle is fenced and the fence EF divides the rectangle in half, find the length of *x* so that the total length of fencing is minimised.

Question 7

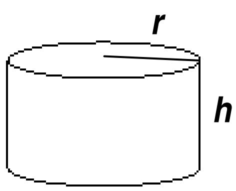
A manufacturing company currently sells 40 machines each week. Their research indicates that for every R2 000 they increase the price of their machines, their sales will drop by 1 machine. The current price of the machines is R72 000 each.

7.1 How many machines need to be sold for the total income to be maximised?

7.2 According to the research, what will be the maximum weekly income?

Question 8

A cylindrical can has a fixed volume, V. Determine the ratio of the height to the radius if the surface area is to be minimised.



Question 1

Use the variables *x* and *y* and write down the equation or inequality that represents each of the following situations. State clearly, in each case, what *x* and *y* represent.

1.1 When mixing green paint, the amount of yellow pigment must be at least double the amount of blue pigment.

1.2 The number of hours spent on Maths and English homework each day should not exceed two.

1.3 The sum of two numbers must be greater than 12.

1.4 The profit made on selling an ice cream must be one-and-a-half times the profit made on selling a chocolate.

1.5 In a hotel, the rooms can accommodate either two people or three people. The total number of guests in the hotel must be less than 250.

1.6 M is the sum of two numbers.

1.7 M is the sum of one number and double another number.

Question 2

Represent each of the following systems of inequalities graphically:

2.1
$$x \le 15$$
$$y \le 12$$
$$x + y \le 20$$

2.2
$$-2 \le x \le 2$$
$$-1 \le y \le 5$$
$$2x + y \le 4$$

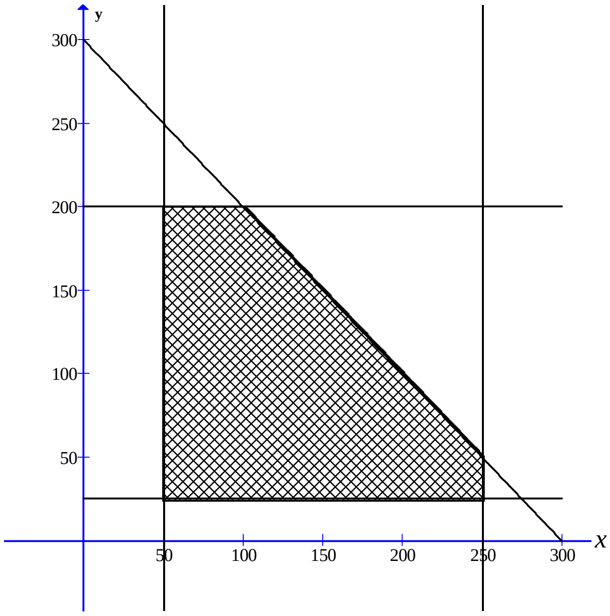
2.3
$$150x + 60y \le 30000$$
$$50x + 60y \le 13000$$
$$10x + 20y \le 5000$$

2.4
$$x \ge 200$$
$$3x + 2y \le 2160$$
$$5x + 2y \le 4800$$
$$\frac{y}{x} \ge \frac{3}{2}$$

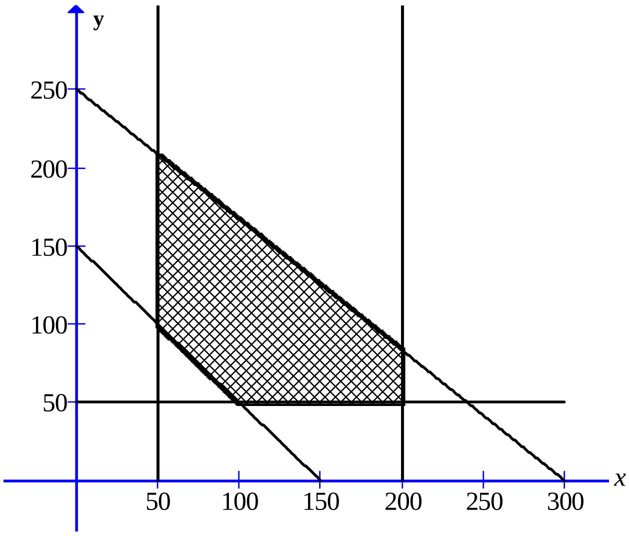
Question 3

Represent each of the following systems of graphs symbolically in terms of *x* and *y*:

3.1



3.2



Question 4

On the grid provided, draw the graph of $2y + x = 12$, where $x \geq 0$ *and* $y \geq 0$

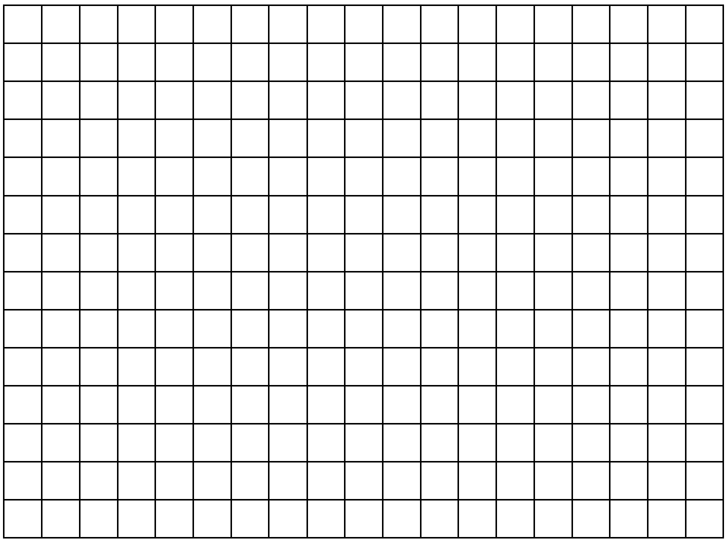
4.1 Give three ordered pairs that are a solution to the equation $2y + x = 12$.

4.2 What is the gradient of the graph?

4.3 State the *x* and *y* intercepts of the graph.

4.4 Using a blue pen, shade the region which represents $2y + x < 12$

4.5 Give three ordered pairs that are a solution to the equation $2y + x < 12$



Question 5

Nomhle and Shaun are playing a game with dice. There is one red dice and one blue dice. A throw (which is a throw of both dice) is awarded a point if it meets the following conditions:

- Neither the red nor the blue dice must be 6.
- The number on the red dice must be greater than the number on the blue dice.
- The sum of the numbers of the two dice must not exceed 7.

5.1 If *x* represents the number on the red dice and *y* represents the number on the blue dice, set up equations that represent the rules of the game as given above.

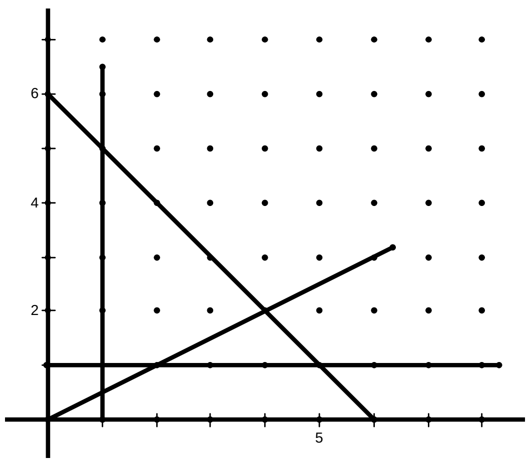
5.2 On graph paper and on the same system of axes, draw the graphs of the equations that you have set up.

5.3 Using your graphs, list all the throws that for which a point is awarded.

5.4 If two points are awarded for an admissible throw where the sum of the two numbers on the dice is a maximum, which throws will be awarded two points?

Question 6

At SmoothSmoothies, the SuperSmoothies contain a base of yoghurt and banana. Customers may choose to add strawberry and melon, subject the following constraints, which have also been expressed mathematically: Using *x* to represent the scoops of strawberry and *y* to represent the scoops of melon, the diagram below represents the linear constraints in making a SuperSmoothie.



a) The added fruit may not exceed six scoops: $(x + y \leq 6)$

b) At least one scoop of each fruit must be added. $(x \geq 1 \;\; and \;\; y \geq 1)$

c) The number of scoops of strawberry may not be more than double the number of scoops of melon. $(x \leq 2y)$

d) No fractions of a scoop are allowed. $(x, y \in Z)$

6.1 Use the letters (a), (b), (c) and (d) to match the constraints to their graphical representation in the diagram above.

6.2 Write down any implicit constraints or constraints that are not represented in the diagram.

6.3 On the diagram, shade the feasible region.

6.4 What is the maximum number of scoops of strawberry in a SuperSmoothie?

6.5 A customer orders a smoothie with 2 scoops of strawberry and 5 scoops of melon. Explain to the customer why it is not possible to have this option in a smoothie.

6.6 If the customer wants 3 scoops of melon, how many scoops of strawberry can the customer have?

6.7 If the customer wants six scoops of fruit, but scoops of strawberry are cheaper than scoops of melon, which is the cheapest option? (i.e. how many scoops of melon and how many scoops of strawberry?)

Question 7

A toy factory makes two different types of wooden toys –coloured blocks (*x*) and mobiles (*y*). According to its worker contracts, the factory guarantees each department a minimum amount of work per day.

The cutting department must have at least 480 minutes of work per day. A set of coloured blocks takes 20 minutes to cut and a mobile takes 10 minutes to cut.

The decorating department must have at least 600 minutes of work per day. A mobile takes 20 minutes to decorate and a set of wooden blocks take 10 minutes to decorate.

The assembly department must have at least 1080 minutes of work per day. A set of coloured blocks takes 10 minutes to assembly and a mobile takes 60 minutes to assemple.

7. If *x* represents the number of set of blocks and *y* represents the number of mobiles, present the information given above as a set of inequalities.

7.2 On graph paper, graph the set of inequalities and indicate the feasible region.

7.3 If materials for a set of blocks costs R15 and the materials for a mobile cost R45, set up an equation which represent the cost (C) of producing paint.

7.4 Plot the cost equation on your graph and use it to determine which vertex will result in the most effective production schedule that minimises costs.

7.5 Find the co-ordinates of this vertex by solving the appropriate two boundary lines.

Question 8

An organic farmer grows lavender and vegetables. He has a plot of land with an area of 9 hectares. In order to fulfil his market contracts the farmer has to plant at least 2 hectares of lavender and 1 hectare of vegetables. In order to minimise the damage done by insects, the area under lavender should be at least equal to the area under vegetables. In order to maintain the nitrogen balance of the soil, the area under lavender should not be more than double the area under vegetables.

8.1 Below is a system of equations that models the scenario described. If *x* is the hectares under lavender and *y* is the hectares under vegetables, extract from the text the words that have resulted in the constraint being modelled (i.e. write down the words from the text that belong to each constraint).

8.1.1 $x \geq y$ 8.1.2 $x \geq 2$ 8.1.3 $x \leq 2y$

8.1.4 $x + y \leq 9$ 8.1.5 $y \geq 1$

8.2 Using graph paper, draw the system of equations and shade the feasible region.

8.3 Set up a Profit equation if the profit on a hectare of lavender is R10 000 and the profit on a hectare of vegetables is R8 000.

8.4 Graph the profit equation and use it to assist you in determining accurately the planting schedule that will result in the farmer maximizing profit.

8.5 If the profit margins changed so that the profit on vegetables and lavender were equal, would it be necessary for the farmer to change his planting schedule? Explain your answer.

Question 9

Creative-cards is a small company that makes two types of cards, Card *X* and Card *Y*. With the available labour and material, the company cannot make more than 150 of Card *X* and not more than 120 of Card *Y* per week. Altogether they cannot make more than 200 cards per week.

There is an order for at least 40 of Card *X* and 10 of Card *Y* cards per week.

Creative-cards makes a profit of R5 for each of Card *X* and R10 for each of Card *Y* .

Let the number of Card *X* manufactured per week be *x* and the number of Card *Y* manufactured per week be *y*.

9.1 State all the constraint inequalities that represent the above situation.

9.2 Represent the constraints graphically and shade the feasible region.

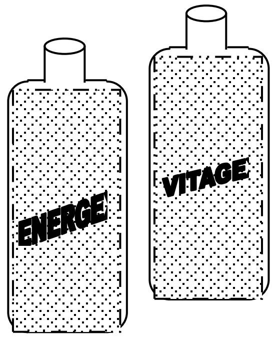
9.3 Write the equation that represents the profit P (the objective function), in terms of *x* and *y*.

9.4 On your graph, draw a straight line which will help you to determine how many of each type of card must be made weekly to produce the maximum profit.

9.5 Calculate the maximum weekly profit.

Question 10

Vitamins B and E are vital ingredients of two types of health drinks, *Energex* and *Vitagex*



- You require 3 g of Vitamin B and 4 g of Vitamin E to produce 1 litre of *Energex*
- You require 9 g of Vitamin B and 6 g of Vitamin E to produce 1 litre of *Vitagex*
- The company has 27 g of Vitamin B and 30 g of Vitamin E per day

  The above information is summarized in the table below:

Ingredients	*Energex*	*Vitagex*	Maximum grams
Vitamin B	3	9	27
Vitamin E	4	6	30

- Furthermore at least 3 litres of *Energex* needs to be produced per day.

Let x and y be the number of litres of *Energex* and *Vitagex* respectively that are produced per day.

10.1 State algebraically, in terms of x and y, the constraints that apply to this problem for a day.

10.2 Represent the constraints graphically on the graph paper provided and shade the feasible region.

10.3 If the profit on 1 ℓ of *Energex* is R30 and the profit on 1 ℓ of *Vitagex* is R50, express the profit, P, in terms of x and y.

10.4 Determine, by making use of a searchline, how many litres of each health drink must be produced in a day to ensure a maximum profit.

10.5 Calculate the maximum possible profit.

Question 1

Determine the equation of the circle in each of the following:

1.1 centre as the origin and radius $\sqrt{3}$

1.2 centre as the origin and (– 4 ; – 3) a point on the circle

1.3 centre (2 ; 3) and radius 6

1.4 centre (1 ; – 7) and radius $\sqrt{5}$

1.5 centre (– 3 ; – 2) the origin and (0 ; – 8) a point on the circle

Question 2

P (12 ; – 5) and Q (–12 ; 5) are the end points of a diameter of a circle.

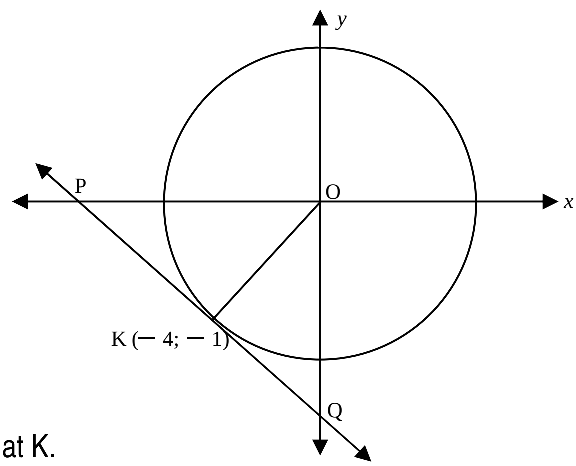
2.1 Prove that the centre of the circle is at the origin.

2.2 Determine the equation of the circle.

2.3 Determine whether the point T (11 ; 6) lies inside, outside or on the circle.

Question 3

In the figure, the circle with centre O passes through the point K(– 4; – 1). The tangent to the circle at K is perpendicular to OK and cuts the *x*-axis at P and the *y*-axis at Q. Determine:



3.1 The equation of the circle

3.2 The equation of the tangent to the circle at K.

3.3 The coordinates of P and Q

Question 4

In a Cartesian plane the equation of a circle with centre M is given by: $x^2 + y^2 + 6y - 7 = 0$

4.1 Determine the co-ordinates of M.

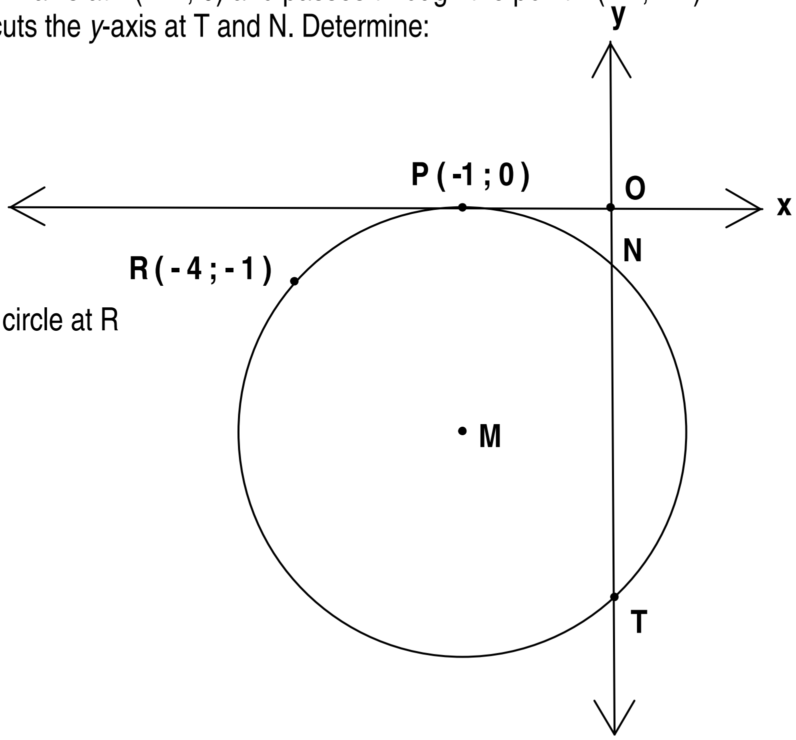
4.2 Determine, by calculation, whether the straight line $y = x + 1$ is a tangent to the circle or not.

Question 5

In the diagram below the circle touches the *x*-axis at P(– 1 ; 0) and passes through the point R(– 4; – 1). M is the centre of the circle and the circle cuts the *y*-axis at T and N. Determine:

5.1 The equation of the circle

5.2 The equation of the tangent to the circle at R



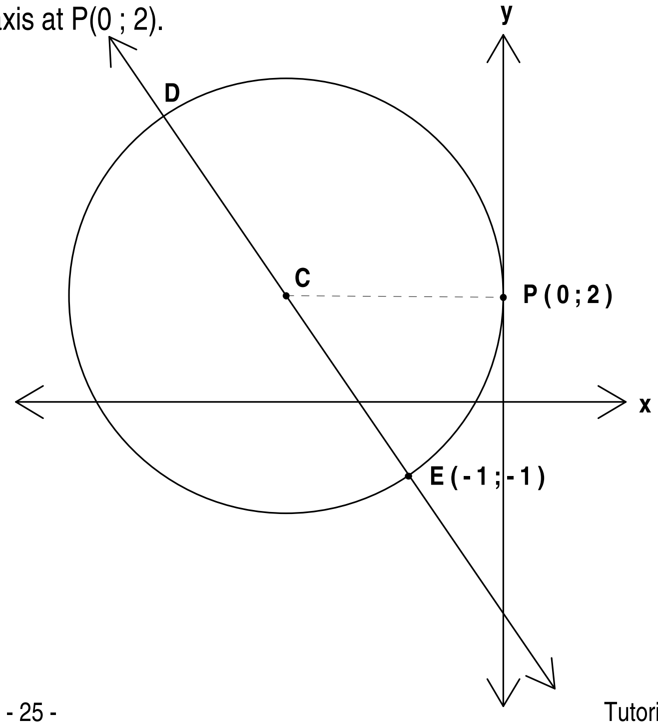
Question 6

In the diagram below, center C of the circle lies on the straight line $3x + 4y + 7 = 0$. The straight line cuts the circle at D and E(–1 ; –1). The circle touches the *y*-axis at P(0 ; 2).

6.1 Determine the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$.

6.2 Determine the length of diameter DE.

6.3 Determine the equation of the tangent to the circle at D



Question 7

The equation of a circle with centre M and radius *r* is $x^2 + y^2 + 4x - 12y + 4 = 0$.

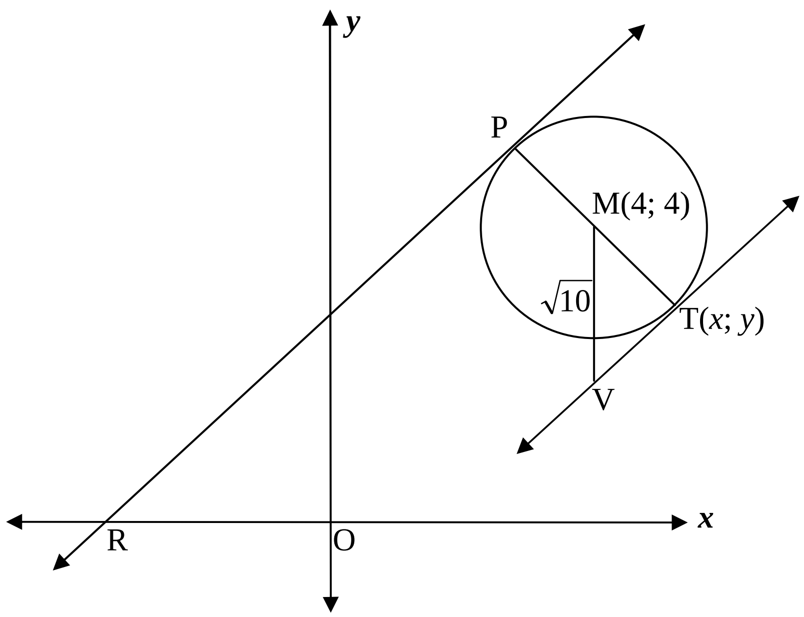
7.1 Calculate the co-ordinates of the centre M and the length of the radius *r*.

7.2 Without any further calculations write down the co-ordinates of the point(s) where this circle intersects the *x*-axis.

7.3 Determine the equation(s) of the tangent(s) to this circle which is parallel to the *y*-axis.

Question 8

In the diagram below the line RP with equation $y - x - 2 = 0$ is a tangent at P to a circle with centre M(4 ; 4). PT is a diameter of the circle.



Determine:

8.1 The equation of PT

8.2 The coordinates of P, the point of contact

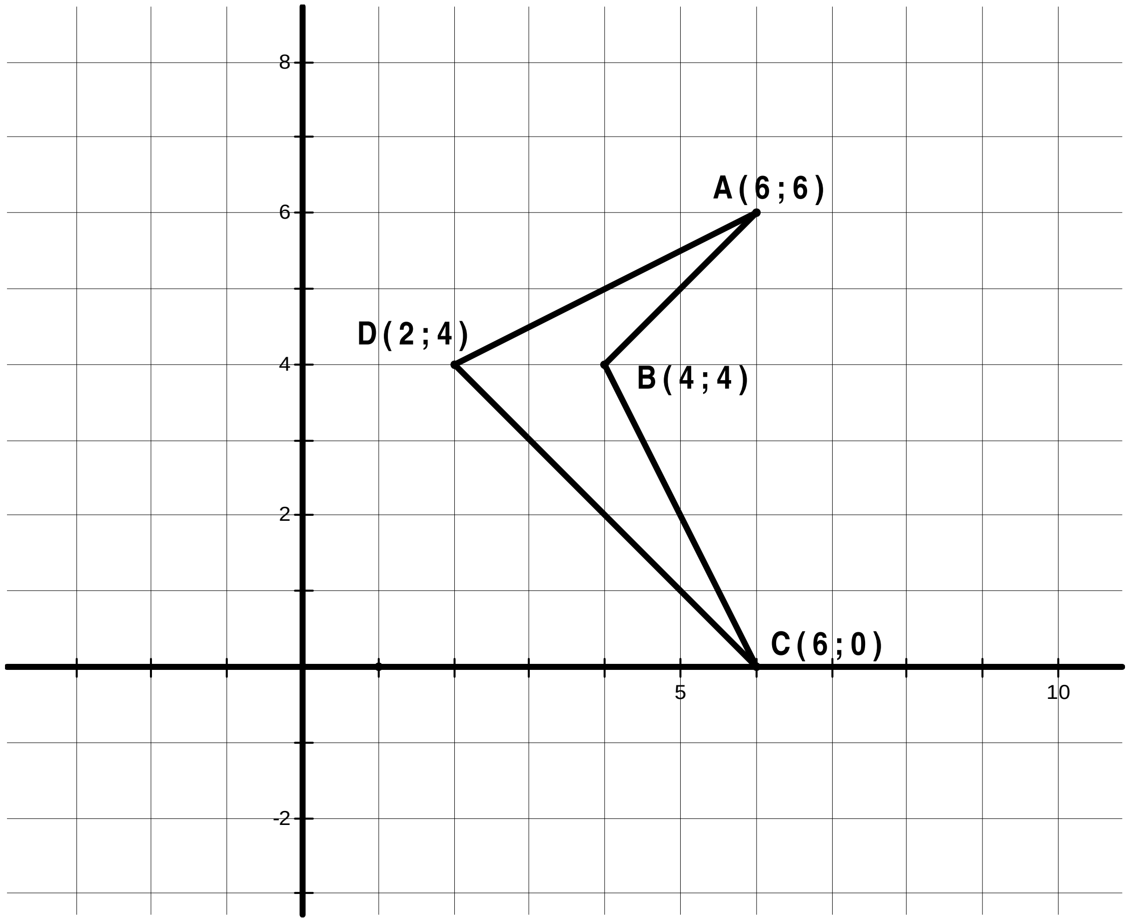
8.3 The equation of the circle

8.4 The coordinates of T

8.5 The length of VT if V is a point on the tangent at T such that MV is $\sqrt{10}$ units from the centre of the circle. Leave the answer in surd form.

Question 1

Using the diagram below, answer the questions that follow.



1.1 ABCD has to be enlarged through the origin by a factor of $\frac{1}{2}$.

1.1.1 Use the grid on the diagram sheet to draw this enlargement and clearly indicate the vertices A'B'C'D'.

1.1.2 Give the coordinates of vertices A' and C' of the enlargement.

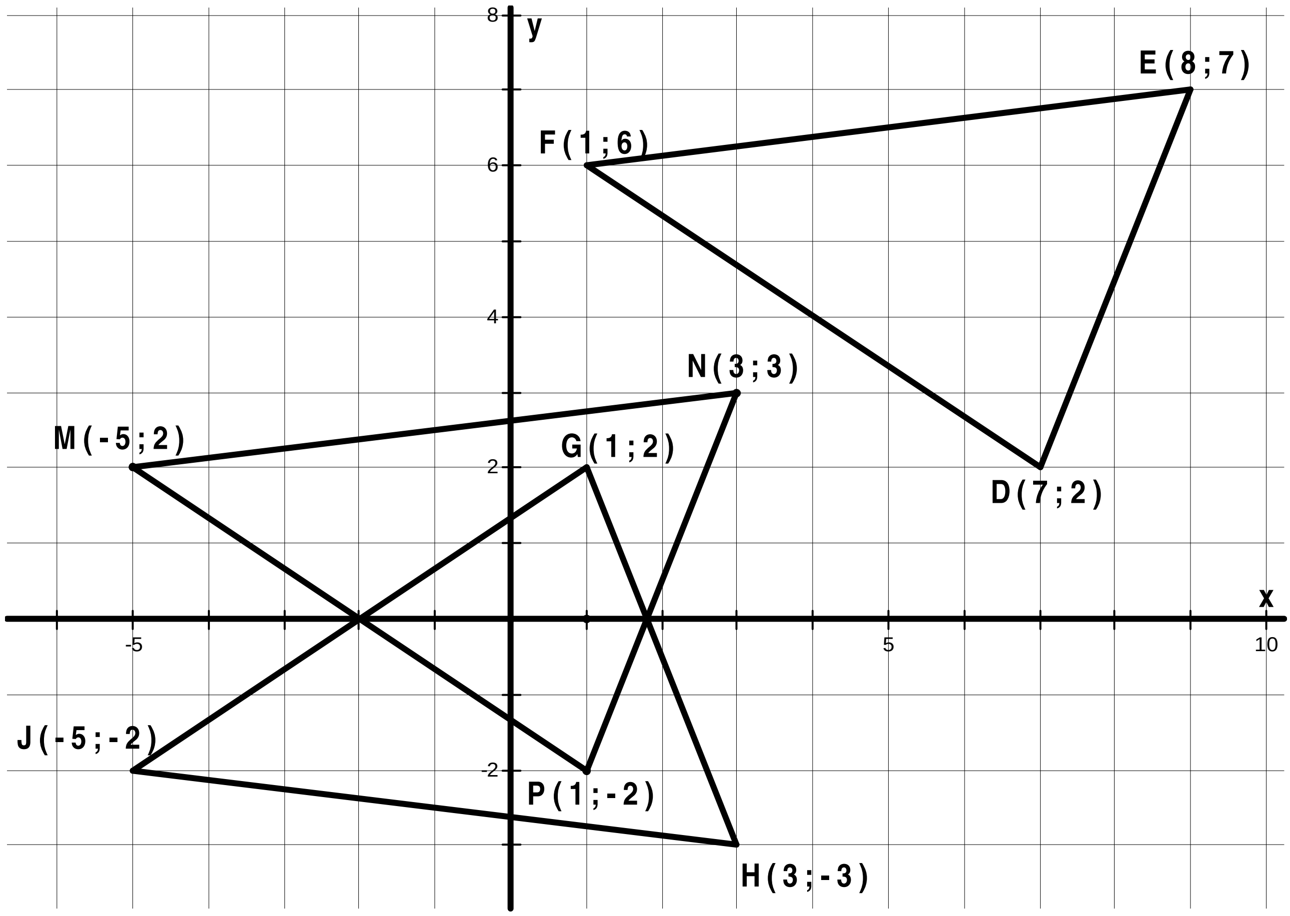
1.1.3 If the area of ABCD is $2x$ square units, determine the area of the enlargement A'B'C'D'.

1.2 Quadrilateral ABCD is rotated 90° in a clockwise direction through the origin.

1.2.1 State the general rule for the coordinates of a point satisfying this type of rotation.

1.2.2 Give the coordinates of the vertices of A"B"C"D" for this rotation.

Question 2



2.1 Describe the transformation in words that would transform Δ PMN to Δ GJH

2.2 Give the general rule for the coordinates of any point undergoing this transformation.

2.3 Give the coordinates of P', if P' is the reflection of P in the line $y = -x$

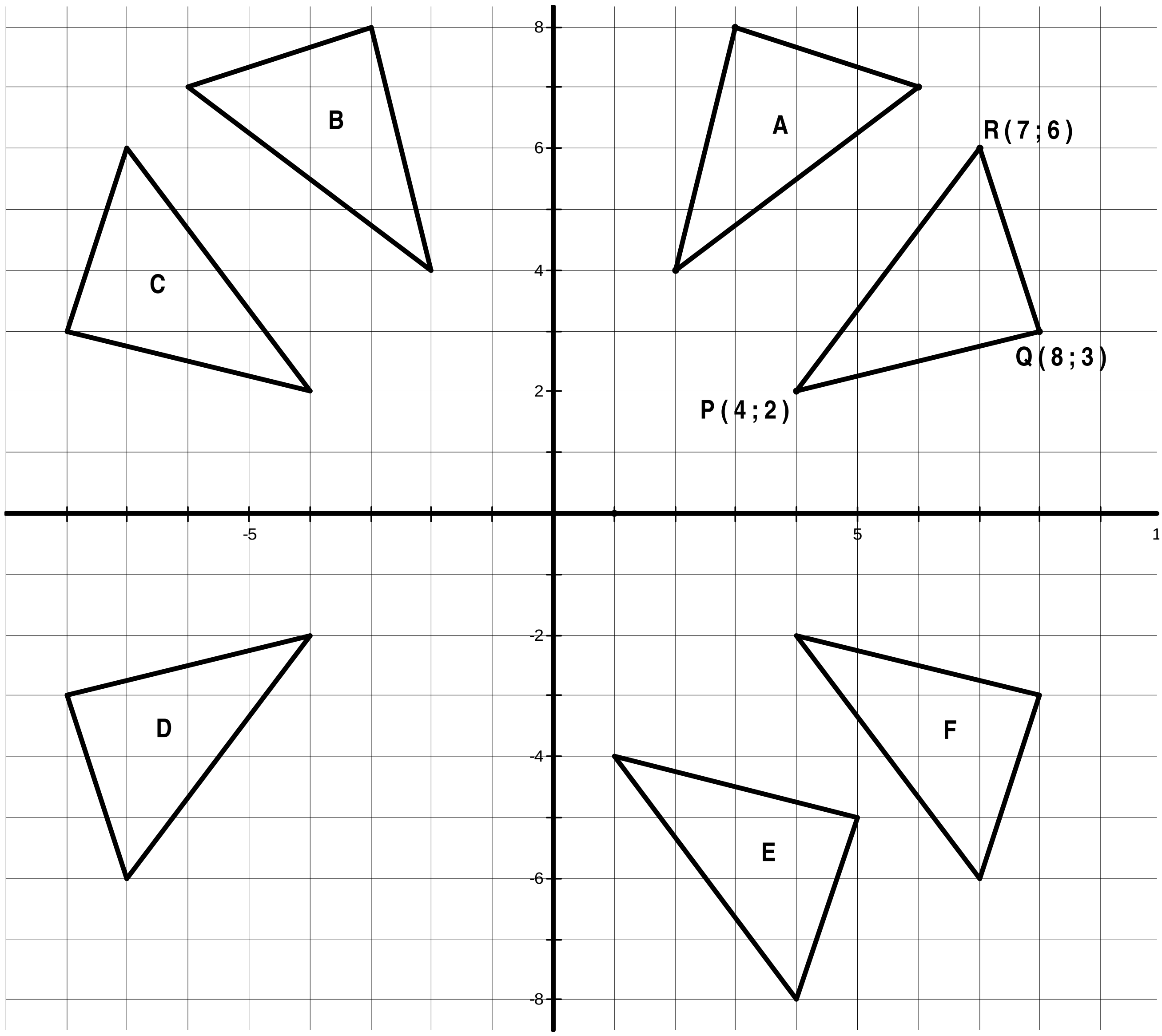
2.4 Describe the transformation in words that would transform Δ PMN to Δ DFE

2.5 Give the general rule for the coordinates of any point undergoing this transformation.

2.6 Write down the coordinates of M' if M is reflected about the x-axis and then reflected about the line $y = x$.

2.7 Give the coordinates of H' if H is first rotated 90° anti-clockwise and then reflected in the y-axis.

Question 3



3.1 Describe, in words, the mapping of $\triangle$PQR to the triangle labelled A

3.2 State the general rule for the coordinates of any point representing the transformation described in 3.1

3.3 Describe two possible transformations that would map triangle PQR to the triangle labelled D

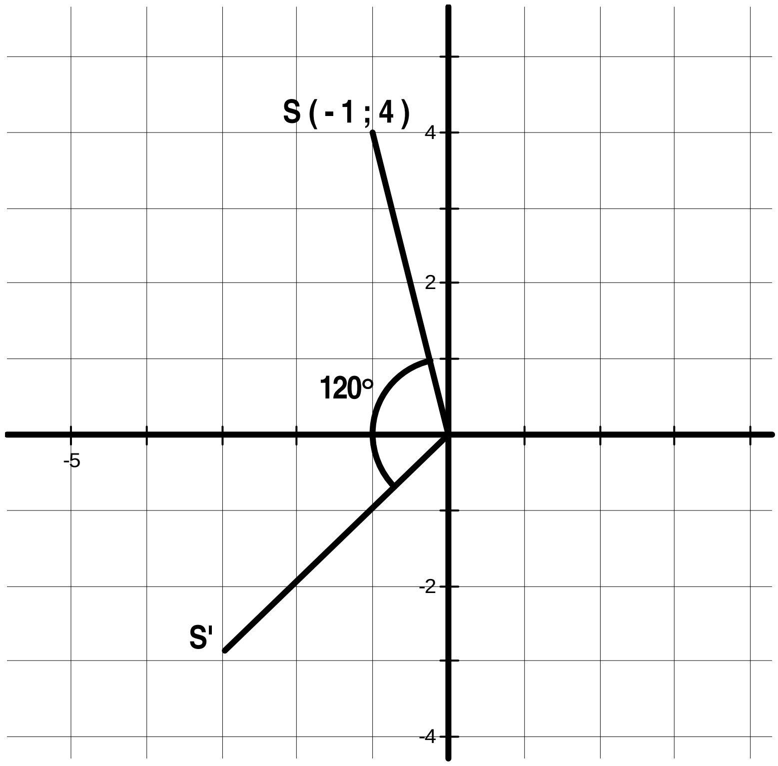
3.4 Onto which triangle would $\triangle$PQR be mapped if it were reflected about the y-axis and then reflected about the line $y = -x$

3.5 Explain what transformations would need to take place in order to map $\triangle$PQR onto $\triangle$E

3.6 Describe how $\triangle$B could be mapped onto $\triangle$D with one transformation

3.7 State the general rule for the coordinates of any point representing the transformation described in 3.6

Question 4



4.1 In the diagram above, S is the point $(-1;4)$. Determine the coordinates of S' if S has rotated 120° about the origin.

4.2 If the point S rotates through an angle of -45° to point S'', determine the coordinates of S''.

Question 1

1.1 On the same system of axes, draw the graphs of $y = \tan\frac{x}{2}$ and $y = 1 - \sin x$ for $x \in [-180^0;180^0]$.

1.2 Use your graphs to determine the approximate values of $x \in [-180^0;180^0]$ for which

$$\tan\frac{x}{2} \geq 1 - \sin x$$

Question 2

Solve the following equations for x. Give the general solution unless otherwise stated. Answers should be given correct to 2 decimal places where exact answers are not possible.

2.1 $2\cos 2x + 1 = 0$ Illustrate you answers on a sketch graph.

2.2 sin $x = 3\cos x$ for $x \in [90^0;360^0]$

2.3 $\sin x = \cos 3x$

2.4 $6 - 10\cos x = 3\sin^2 x$ for $x \in [-360^0;360^0]$

2.5 $2 - \sin x\cos x - 3\cos^2 x = 0$

2.6 $3\sin^2 x - 8\sin x + 16\sin x.\cos x - 6\cos x + 3\cos^2 x = 0$

Question 3

Prove the following identities, stating any values of the variable for which the identity is not valid:

3.1 $\cos x + \tan x \sin x = \dfrac{1}{\cos x}$

3.2 $\dfrac{\sin\theta}{1-\cos\theta} - \dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\sin\theta}$

3.3 $\dfrac{1-\cos^2 x}{\cos x} = \tan x \sin x$

3.4 $\dfrac{\sin^3 x + \sin x\cos^2 x}{\cos x} = \tan x$

3.5 $\dfrac{1+\tan x}{1-\tan x} = \dfrac{1+2\sin x\cos x}{\cos^2 x - \sin^2 x}$

Question 4

Simplify:

4.1 $\dfrac{\sin(180^0 - x)\tan(-x)}{\tan(180^0 + x)\cos(x - 90^0)}$

4.2 $\dfrac{\sin(180^0 + x)\tan(x - 360^0)}{\tan(360^0 - x)\cos 240^0 \tan 225^0}$ (without using a calculator)

Question 5

Given that sin 17^0 = *a*, express in terms of *a*:

5.1 $\cos 73^0$

5.2 $\cos(-163^0)$

5.3 $\tan 197^0$

5.4 $\cos 326^0$

Question 6

Given that 5 cos x +4 = 0, calculate, without the use of a calculator, the value(s) of

6.1 5 sin x + 3 tan x

6.2 $\tan 2x$

Question 7

Given that $y = f(x) = 2\cos x$ and $y = g(x) = \sin(x + 30^0)$:

7.1 Sketch the graphs of *f* and *g* on the same set of axes for $x \in [-180^0; 180^0]$

7.2 Read the following answers from your graph and show where these answers have been read:

7.2.1 Give the period of *f*.

7.2.2 Determine one value of *x* for which $f(x) - g(x) = 1{,}5$

7.2.3 Determine the positive values of *x* for which

$$2\sin(x + 30^0)\cos x < 0$$

7.2.4 Calculate the general solution of the equation $f(x) = g(x)$ and hence write down the co-ordinates of the two points of intersection of $f(x)$ and $g(x)$ between -180^0 and 180^0

7.2.5 If the curve of f were moved down by 1 unit, what would the equation be?

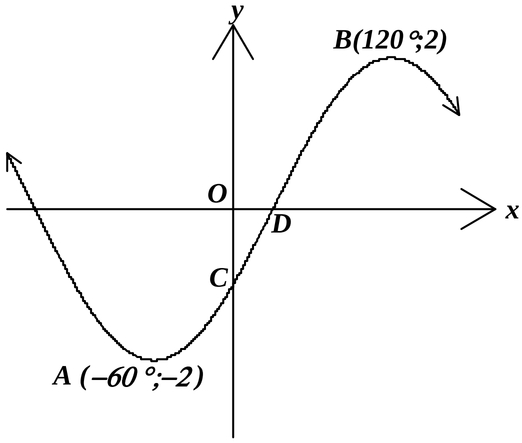
Question 8

Sketched is the graph of $y = a\cos(x+b)$. A and B are turning points.

Calculate the values of:

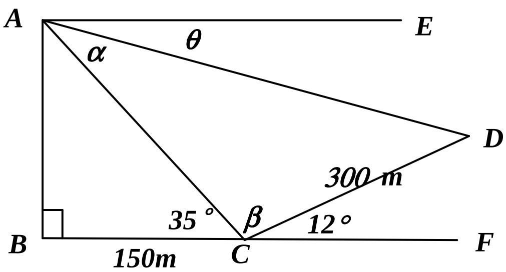
8.1 *a* and b

8.2 the coordinates of C and D.



Question 9

Standing at C, 150 m from the bottom, B, of a vertical cliff AB, Thabo observes that the angle of elevation of A, the top of the cliff, is 35^0. He then walks 300 m to the point D, up a slope inclined at 12^0 to the horizontal. A, B, C, D and E are in the same vertical plane.



Calculate

9.1 the size of β.

9.2 the distance AC

9.3 the size of α

9.4 the angle of depression, θ, of D from A, the top of the cliff.

Question 10

In ΔABC, sketched below, D is a point on BC such that BD = 7 units, and DC = 5 units. AB = 13 units.

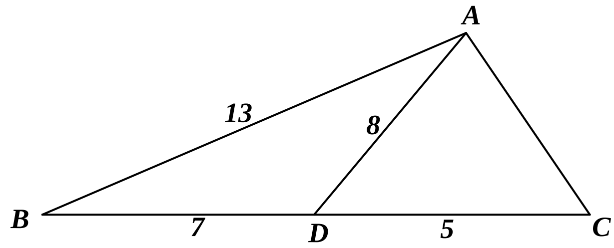
Calculate, without using a calculator:

10.1 the size of $B\hat{D}A$

10.2 the length of AC

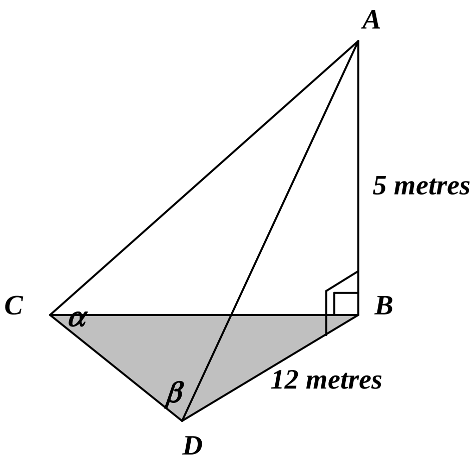
and hence or otherwise:

10.3 show that the area of $\Delta ABC = 24\sqrt{3}$ square units.



Question 11

In the given diagram, AB is a vertical flagpole 5 metres high. AC and AD are two stays with B, C and D in the same horizontal plane. BD = 12 metres, $A\hat{C}D = \alpha$ and $A\hat{D}C = \beta$.

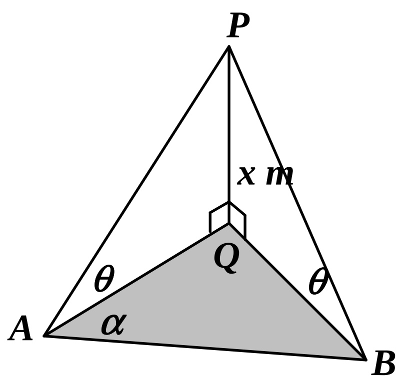


Show that the distance $CD = \dfrac{13\sin(\alpha+\beta)}{\sin\alpha}$

Question 12

At a point A at sea, a sailor records that the angle of elevation of P, the top of a lighthouse PQ is θ. The lighthouse is due north of the position of the boat. The sailor then sails to B on a bearing of α^0 and then notices that the angle of elevation of P is again θ. The top of the lighthouse is x metres above the sea.

Show that the distance $AB = \dfrac{2x\cos\alpha}{\tan\theta}$



Question 1

Fifteen households were surveyed in Saldanha, Oudtshoorn and Stellenbosch with regard to the monthly amount spent on electricity. The results are recorded in the table below:

Saldanha	Oudtshoorn	Stellenbosch
R 52.00	R 103.00	R 246.00
R 112.00	R 92.00	R 126.00
R 83.00	R 67.00	R 226.00
R 256.00	R 140.00	R 101.00
R 412.00	R 136.00	R 92.00
R 61.00	R 183.00	R 67.00
R 54.00	R 214.00	R 63.00
R 81.00	R 87.00	R 71.00
R 147.00	R 145.00	R 167.00
R 134.00	R 135.00	R 129.00
R 61.00	R 164.00	R 117.00
R 78.00	R 103.00	R 135.00
R 225.00	R 129.00	R 129.00
R 23.00	R 152.00	R 168.00
R 189.00	R 118.00	R 131.00

1.1 Calculate the following for each of the municipalities:

1.1.1 The mean
1.1.2 The median

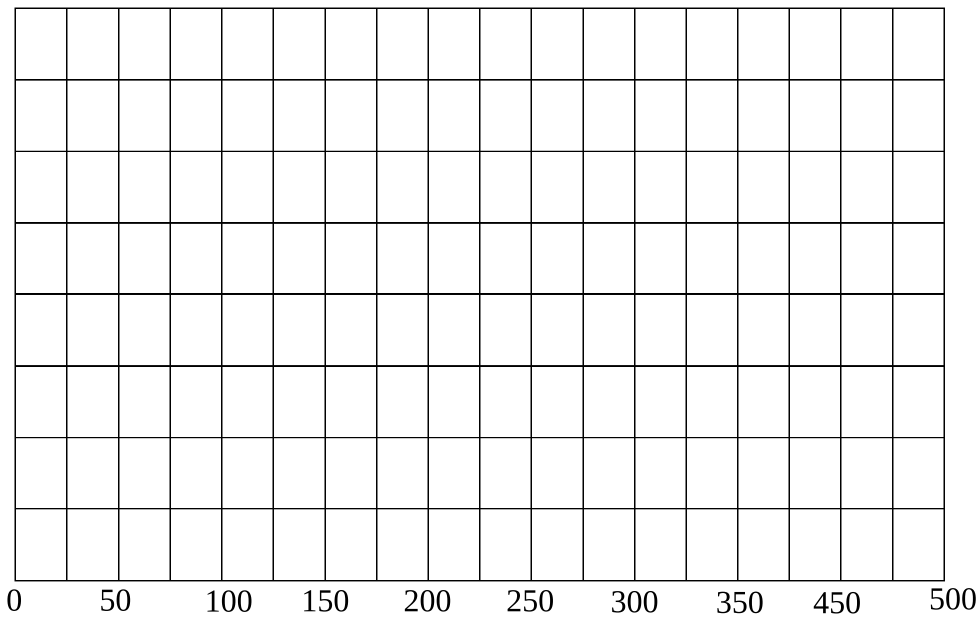
1.2 Comment on how these measures are similar or vary in the three sets of data.

1.3 Explain why the mode is not a suitable measure to describe any of the sets of data.

1.4 Calculate the following for each of the municipalities:

1.4.1 The maximum value
1.4.2 The minimum value
1.4.3 The range
1.4.4 The upper quartile
1.4.5 The lower quartile
1.4.6 The interquartile range

1.5 Using the 5 number summary for each of the sets of data, draw box and whisker diagrams for each of the municipalities. Use the grid provided below:



1.6 Using any of the statistical measurements you have calculated, as well as the box and whisker diagrams, write a short paragraph comparing the household expenditure on electricity in Saldanha, Oudtshoorn and Stellenbosch.

1.7 The table below gives grouped data for the household electricity expenditure in Saldanha, Oudtshoorn and Stellenbosch.

Expenditure in R	Number of households		
	Saldanha	Oudtshoorn	Stellenbosch
<50	1	0	1
50 - 99	7	1	2
100 - 149	2	8	4
150-199	1	5	5
200-249	1	1	2
250-299	1	0	1
300-349	0	0	0
350-399	1	0	0
400+	1	0	0

1.7.1 Using the table above and your answers to question 1.1, complete the following table:

	Modal class	Median	Mean
Saldanha			
Oudtshoorn			
Stellenbosch			

1.7.2 Explain why, when the data has been grouped, it has become appropriate to use the mode (or modal class) to describe the data.

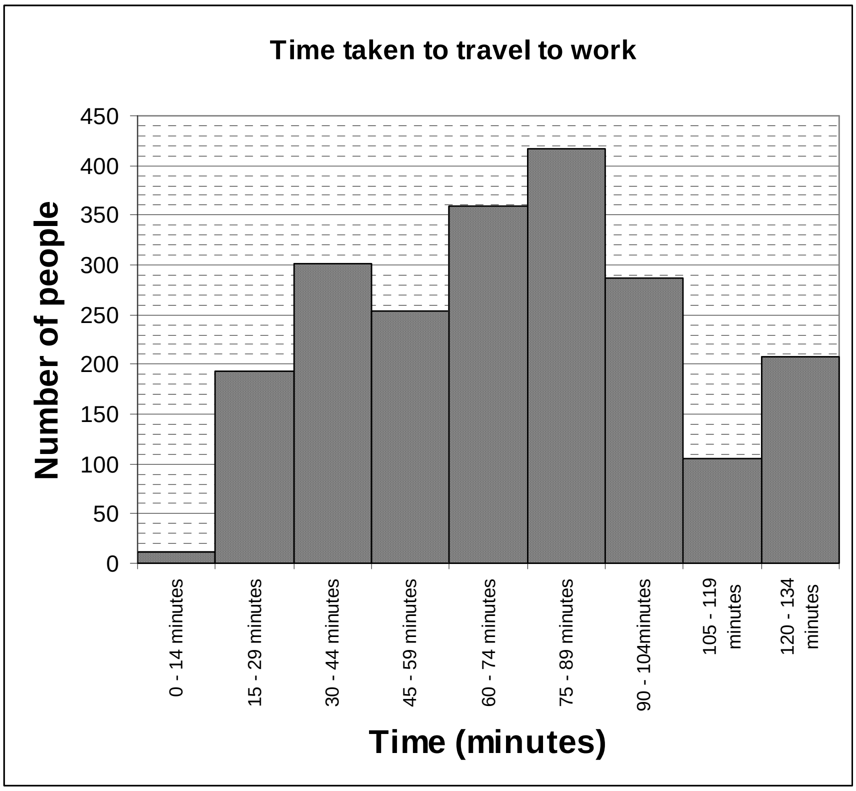
1.7.3 Draw three separate histograms (using the table in 1.7 above) to represent the electricity expenditure for Saldanha, Oudtshoorn and Stellenbosch.

1.7.4 Mark the mean and the median on each histogram.

1.7.5 Using the histograms, write a short comment on electricity expenditure patterns in the three municipalities.

Question 2

A group of people were surveyed with respect to how long it took them to travel to work each day. The results of the survey are summarized in the chart below.



2.1 What is the range?

2.2 What is the modal class?

2.3 Which class contains the median?

2.4 Calculate the estimated mean. Show your calculations and work correct to the nearest minute.

Question 3

Oudtshoorn Monthly Household Income (R)	Oudtshoorn Monthly Electricity Expenditure (R)
R 5,534.00	R 92.00
R 5,886.00	R 103.00
R 6,231.00	R 107.00
R 6,671.00	R 139.00
R 7,004.00	R 118.00
R 7,421.00	R 135.00
R 7,821.00	R 140.00
R 7,974.00	R 145.00
R 8,023.00	R 146.00
R 8,368.00	R 164.00
R 8,541.00	R 152.00
R 8,718.00	R 168.00
R 9,687.00	R 177.00
R 10,355.00	R 183.00
R 12,076.00	R 214.00

The table alongside gives the monthly household income and monthly electricity expenditure for 15 families in Oudsthoorn.

3.1 Calculate the mean monthly household income.

3.2 Complete the table below and then calculate the standard deviation of the monthly household income.

3.3 Calculate the percentage household incomes that fall within one standard deviation of the mean.

3.4 Use the data in the table at the beginning of question 2 to draw a scatterplot of monthly household income against monthly electricity expenditure.

3.4.1 Draw a line of best fit on your graph.

3.4.2 Is the relationship between household income and electricity expenditure linear or exponential? Justify your answer.

3.4.3 Use your graph to predict the electricity expenditure in a household with an income of R3500 per month.

Income (R)	$(x_i - \overline{x})$	$(x_i - \overline{x})^2$
R 5,534.00		
R 5,886.00		
R 6,231.00		
R 6,671.00		
R 7,004.00		
R 7,421.00		
R 7,821.00		
R 7,974.00		
R 8,023.00		
R 8,368.00		
R 8,541.00		
R 8,718.00		
R 9,687.00		
R 10,355.00		
R 12,076.00		
$\sum_{i=1}^{n}(x_i - \overline{x})^2$		

3.4.4 The electricity supplier has requested that residents in Saldanha cut their electricity consumption by 10%. Assuming the residents cooperate with this request, how will the line of best fit be affected?

Question 4

The housing department recently conducted a survey on the number of bedrooms in 200 houses in Saldanha. The following data was obtained:

No. of bedrooms	1	2	3	4	5	6	7	8	9	10
No. of houses	25	40	42	28	22	18	12	7	4	2

4.1 Set up a cumulative frequency table for the given data.

4.2 Hence draw an ogive (cumulative frequency graph) for this data.

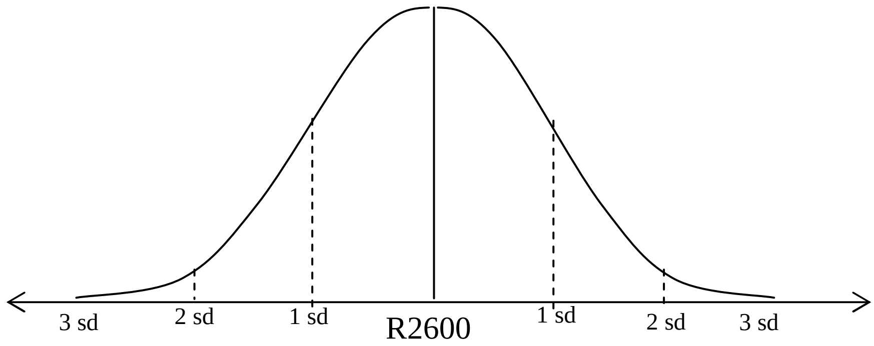
4.3 Use the ogive to determine the median, lower quartile (Q_1) and upper quartile (Q_3).

4.4 Draw a box and whisker diagram for this data.

4.5 What is the value of the interquartile and semi-interquartile range?

Question 5

The monthly household income for families in Oudtshoorn is represented in the graph below.



The results suggest that the household income is symmetrically distributed with a mean of R2600 per month and a standard deviation of R950 per month. Research has suggested that if the monthly household income is below R1650 the family's income would be deemed to be below the poverty line.
It is also known that :

68% of the monthly household income recorded is within one standard deviation of the mean: 34% above and 34% below;

96% are within two standard deviations of the mean: 48% above and 48% below ;

100% are within three standard deviations of the mean: 50% above and 50% below.

Estimate the percentage of families:

5.1 Whose income is below the poverty line

5.2 Earning more than R3550 per month

5.3 Earning less than R700 per month.