# Grade 12

# Tutorial Guide

# 2008

## GRADE 12 TUTORIAL GUIDE

1.1 2; 5; 8; 11; …

$5-2=8-5=3$

$\therefore d=3$

$T_{250}=a+249d=2+249(3)=749$

1.2 $2+(n-1)(3)=302$

$\therefore n=101$

302 is the 101th term

1.3 $610=\frac{n}{2}[2(2)+(n-1)(3)]$

$\therefore 1\,220=n+3n^2$

$\therefore 0=3n^2+n-1\,220$

$\therefore 0=(3n+61)(n-20)$

$\therefore n=-\frac{61}{3} \quad or \quad n=20$

Hence $S_{20}=610$

2. $ar^3=250$ and $ar^5=6\,250$

$\therefore \frac{ar^5}{ar^3}=\frac{6\,250}{250}=25$

Since the terms are positive $r=5$ and $a=2$

3.1.1 $a=81$ and $81+3d=3$

$\therefore d=-26$, $x=55$ and $y=29$

3.1.2 $a=81$ and $81r^3=3$

$\therefore r=\frac{1}{3}$, $x=27$ and $y=9$

3.2 $S_\infty=\frac{81}{1-\frac{1}{3}}=\frac{243}{2}$ (or $121\frac{1}{2}$)

4. $T_n=-239=6+(n-1)(-5)$

$\therefore 5n=11+239$

$\therefore n=125$

$\therefore S_{125}=\frac{125}{2}[2(6)+124(-5)]$

$=-38\,000$

5. $a=\frac{1}{6}, \quad r=2$

$\therefore \frac{1}{6}(2)^{n-1}=\frac{256}{3}$

$\therefore 2^{n-1}=2^9$

Hence $T_{10} = \dfrac{256}{3}$

6. $\sum_{k=1}^{40} 5.2^{k-1} = 5 + 10 + 20 + ... + 5.2^{39}$

$S_{40} = \dfrac{5[2^{40} - 1]}{2 - 1} = 5.497558139 \times 10^{12} \approx 5\,497\,558\,139\,000$

We know this is only approximate because the actual sum would end in a 5.
Needless to say this is a *very* large number and the best the calculator can manage is a number correct to the nearest 1 000.

7. The first circle has an area of $4\pi$ and hence the radius is 2 and the diameter is 4.

The second circle has an area of $\dfrac{9}{4} \times 4\pi$ and hence the radius is 3 and the diameter 6.

The diameters form a geometric sequence with a common ratio of $\dfrac{3}{2}$

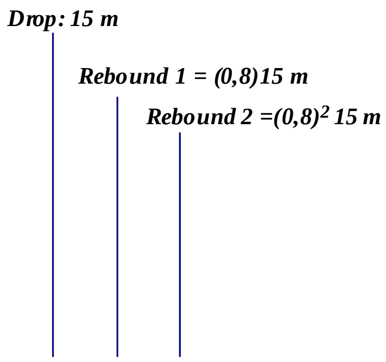
So $4 + 6 + 9 + ... = \dfrac{665}{8}$

$\therefore \dfrac{4\left[\left(\frac{3}{2}\right)^n - 1\right]}{\frac{3}{2} - 1} = \dfrac{665}{8}$

$\therefore \left(\dfrac{3}{2}\right)^n = \dfrac{665}{8} \times \dfrac{1}{8} + 1 = \dfrac{729}{64}$

$\therefore n = 6$ and there are six circles.

8. The first drop is 15 metres, the first rebound is $0{,}8 \times 15$ metres high…




The rebounds form a geometric sequence $(0{,}8)15; \quad (0{,}8)^2 15; \quad (0{,}8)^3 15 : ...$
The total distance travelled by the ball is

$$15 + 2 \times (0{,}8)15 + 2 \times (0{,}8)^2 15 + 2 \times (0{,}8)^3 15 + ...$$

8.1 Second rebound = $(0{,}8)^2 \times 15 = 9{,}6\ m$

8.2 $(0{,}8)^n \times 15 < 3$

$\therefore 0{,}8^n < 0{,}2$

$\therefore n \log 0{,}8 < \log 0{,}2$

We divide both sides by log 0,8 (which is negative)
so we reverse the direction of the inequality sign.

$$\therefore n > \frac{\log 0{,}2}{\log 0{,}8} = 7{,}2...$$

The eighth rebound will be lower than 3 metres.

7 bounces are higher than 3 metres.

8.3 Total distance travelled = 15 $+ S_\infty$ where $a = 2 \times 0{,}8 \times 15 = 24$ and $r = 0{,}8$

$$= 15 + \frac{24}{1 - 0{,}8} = 135m$$

9. $2\ 500 + (n-1)750 = 4\ 000 + (n-1)500$

$$\therefore 750n - 500n = 1\ 500 + 250$$

$$\therefore n = 7$$

In the eighth year Lindiwe's salary exceeds Christina's

10.1 $S_5 = 0{,}2(0{,}2)^2 + 0{,}2(0{,}4)^2 + 0{,}2(0{,}6)^2 + 0{,}2(0{,}8)^2 + 0{,}2(1)^2 = 0{,}44$ square units

10.2 $A = \frac{1}{n}\left(\frac{1}{n}\right)^2 + \frac{1}{n}\left(\frac{2}{n}\right)^2 + \frac{1}{n}\left(\frac{3}{n}\right)^2 + ... + \frac{1}{n}\left(\frac{n}{n}\right)^2 = \frac{1}{n^3}\left[1^2 + 2^2 + 3^2 + ... + n^2\right]$

$$= \frac{1}{n^3}\sum_{k=1}^{n} k^2$$

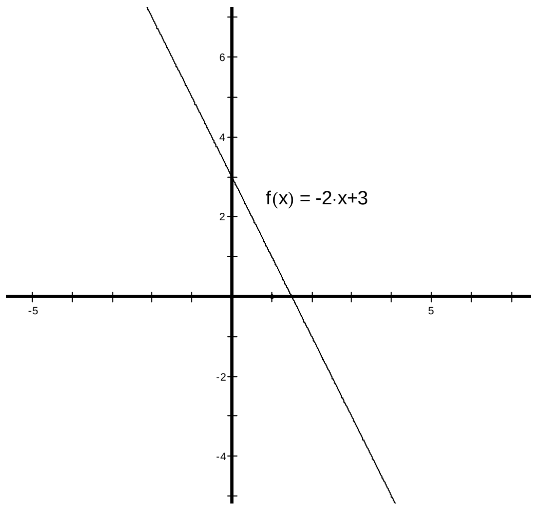
10.3 Area of 100 rectangles = $\frac{1}{100^3}\left[\frac{100(101)(201)}{6}\right] = 0{,}33835$ square units

10.4 0,33835 is closer as the sections of the rectangles which go above the curve $y = x^2$ get smaller as the number of rectangles gets bigger.

1.1 A function is a relationship between two sets of variables, the domain and the range, where each element of the domain is associated with only one element of the range.

1.2 $f(-1) = -2(-1) + 3 = 2 + 3 = 5$

1.3




1.4 If any vertical line drawn through the graph cuts the graphs only once, then the graph represents a function.

1.5 If any horizontal line drawn through the graph cuts the graphs only once, then the graph represents a one-to-one function.

2.1 $f(1) = \pm\sqrt{16} - 1$
$f(1) = 3 \; or \; -5$

2.2 $x = 1$ maps onto both 3 and -5, so *f* is not a function

2.3 It would not be a function. Any vertical line drawn through the graph would still cut the graph in two places.

3.1 Function; one-to-one.

3.2 Not a function; either $y \leq 0$ or $y \geq 0$.

3.3 Function; one-to-one.

3.4 Function; many-to-one.

3.5 Not a function; either $y \leq 0$ or $y \geq 0$.

3.6 Not a function; either $y \leq 1$ or $y \geq 1$.

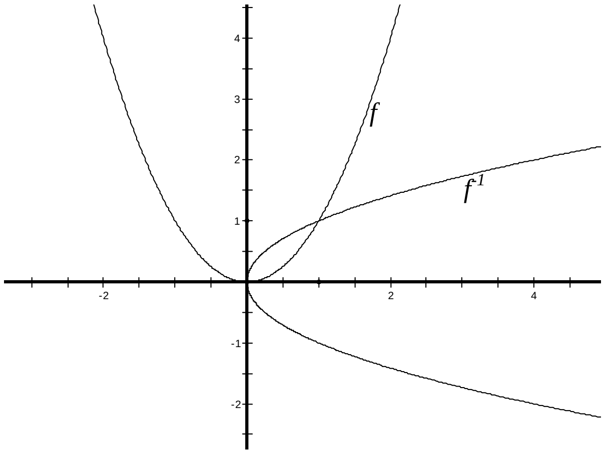
4.1 Not a one-to-one mapping. Every value of *x* will map onto 5.

4.1 One-to-one. Each value of the domain maps onto a unique value of the range.

4.3 One-to-one. Each value of the domain maps onto a unique value of the range.

4.4 Not a one-to-one mapping. $\sin 5(0°) = \sin 5(36°) = 0$.

5.1, 5.2



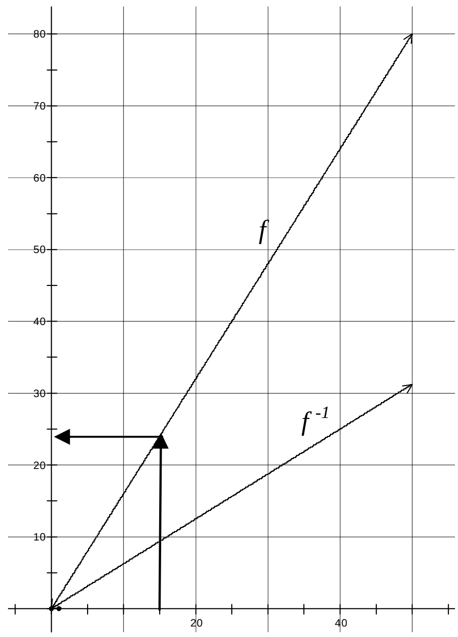

5.3 $f^{-1}(x) = \pm\sqrt{x}$

5.4 Each value of the domain ( $x \geq 0$) maps onto two values of the range. Any vertical line does not cut the graph once only.

5.5 Either $x \leq 0$ or $x \geq 0$.

6.1 Yes. Each mile value (*x*) maps onto a unique km (*y*) value.

6.2




6.3 24Km

6.4 $f^{-1}(x) = \dfrac{5}{8}x$

6.5 The conversion from Km (*x*-axis) to miles (*y*-axis)

6.6 *f*: (10; 16) $f^{-1}$: (16;10); *f*: (25; 40) $f^{-1}$: (40;25); shown on the graph. *x* and *y* have been interchanged.

7.1 (f)

7.2 (g)

7.3 (c)

7.4 (b)

7.5 (a) $f(x) = -5^{-x}$

(b) $f(x) = \log_5 x$

(c) $f(x) = -\log_5 x$

(d) $f(x) = -\log_5(-x)$

(e) $f(x) = \log_5(-x)$

(f) $f(x) = 5^x$

(g) $f(x) = 5^{-x}$

(h) $f(x) = -5^x$

8.1 2

8.2 On graph

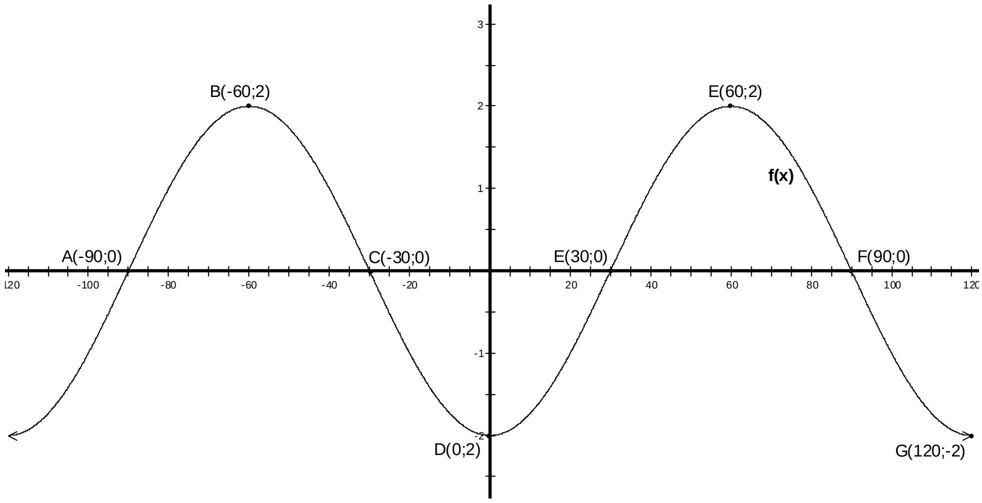
8.3 $-2\cos 3x = -2\cos(-3x)$

Therefore, if *x* is replaced with –*x*, the value of *f*(*x*) remains unchanged and so *f* is symmetrical about the *y*-axis.

8.4 $f(x) = -2\cos 3(x + 15°)$

8.5 Graph will move up one unit - $f(x) - 1 = -2\cos 3x;\quad f(x) = -2\cos 3x + 1$

Range: $-1 \leq f(x) \leq 3$




9.1 $2 = a(2)^2$, $a = \frac{1}{2}$; $2 = \frac{k}{2} + 1$, $k = 2$

9.2 (0;0) and $x = 0$

9.3 $(-2;2)$ symmetry about line $x = 0$, therefore replace *x* with –*x* in point (2; 2)

9.4 $x = 0$ and $y = 1$

9.5 $x = -2$

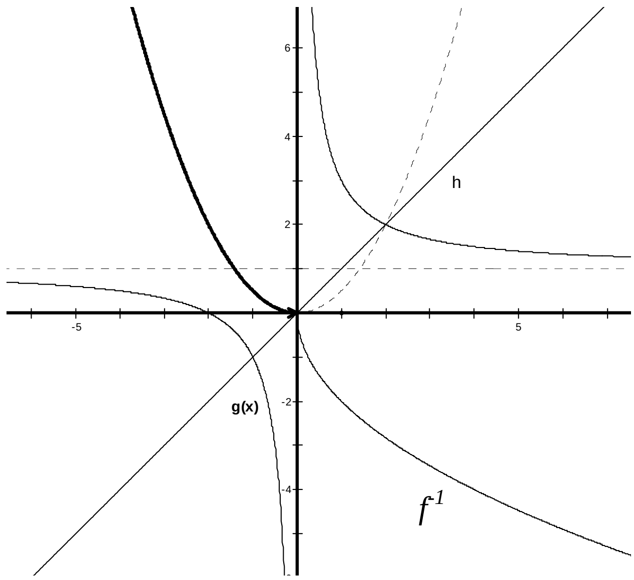
9.6 $x > 0$

9.7 On graph

9.8 If (-2;0) lies on line, so should (0;-2) for *y*=*x* symmetry.

$g(0) = \frac{2}{0} + 1,$

Undefined, therefore the symmetry does not exist.




9.9 On graph

9.10 (2;-2)

9.11 $y \leq 0$

9.12 Inverse will shift left (negative horizontal shift) 2 units.

1.1 $x = 0$ or $x = -6$

1.2 $x = 2$ or $x = 5$

1.3 $x = 1{,}12$ or $x = -1{,}79$ (to 2 dec) or $\frac{-1 \pm \sqrt{19}}{3}$

1.4 $1 < x < 2$

1.5 $x = 8$

1.6 $x = 5{,}12$

1.7 $x = -4$ or $x = -6$

1.8 $x = -6$

1.9 $x > \frac{1}{9}$

1.10 $x = -1$ or $x = 1$

1.11 $1 < x < 9$

1.12 No solution

1.13 $x = -1$ or $x = -\frac{3}{2}$ or $x = 8$

1.14 $x = 3{,}9$

1.15 $x = 1{,}45$

1.16 $x = -2\sqrt{2}$

2.1 $x = 0$ or $x = 8$

2.2 $k = \pm 1$ or $k = \pm 3$

3.1 $y \in R$

3.2 $y = 3$

4. $x \neq 0,$ $x > -\frac{5}{2}$

5 units digit = 8 because $2^{1999} \times 3^{2000} = \frac{2^{2000} \times 3^{2000}}{2} = \frac{6^{2000}}{2}$ and since all powers of 6 end with an odd tens digit and a units digit of 6, half of this will always end in an 8

6.1 $x = -\frac{1}{5}$ and $y = \frac{7}{5}$ or $x = 1$ and $y = -1$

6.2 $x = -\frac{5}{9}$ and $y = \frac{7}{9}$ or $x = 3$ and $y = -1$

6.3 $x = 1$ and $y = 2$ or $x = 2$ and $y = 1$

6.4 $x = 1$ and $y = 0$ or $x = 4$ and $y = 6$

7.1 Surface Area = $4xy + 2x^2 = 288$
$\therefore \quad 2x^2 + 4xy - 288 = 0$
$\therefore \quad x^2 + 2xy - 144 = 0$

7.2 When $x = 6$, $y = 9$
When $x = 8$, $y = 5$

8 Let number of workers be $x$
Let daily wage per worker be $y$

$\therefore \quad xy = 5880$

$\therefore \quad x = \frac{5880}{y}$

$$(x - 4)(y + 10) = 5850$$
$$xy + 10x - 4y - 40 = 5850$$
$$\frac{5880}{y}(y) + 10\left(\frac{5880}{y}\right) - 4y - 40 = 5850$$
$$58800 - 4y^2 = 10y$$
$$2y^2 + 5y - 29400 = 0$$
$$y = 120 \quad \text{or} \quad y = -\frac{490}{4} \text{ (which is not valid)}$$

$\therefore \quad y = 120$
$\therefore \quad x = 49$

9.1 $(x - 2)(x + 3)(x - 5)$

9.2 $(-x + 1)(x - 1)(x - 7)$

9.3 $(x + 1)(x + 1)(x + 1)$

9.4 $(x - 1)(x - 1)(x - 3)$

9.5 $(x + 1)(2x - 1)(x - 3)$

1.1 $50\ 000 = 200\ 000(1-0{,}16n) \quad \therefore 0{,}16n = 1-\frac{1}{4} \quad \therefore n = 4{,}6875$

It will take4 years and 8 months (to nearest month)

1.2 $50\ 000 = 200\ 000(0{,}84)^n \quad \therefore n = \frac{\log 0{,}25}{\log 0{,}84} = 7{,}951...$

It will take 7 years 11 months (to nearest month)

2. $2\ 400 = 1\ 500(1+5i) \quad \therefore i = 12\%$

3.A= $12\ 000\left(1+\frac{0{,}112}{12}\right)^2\left(1+\frac{0{,}115}{12}\right)+12\ 000\left(1+\frac{0{,}112}{12}\right)\left(1+\frac{0{,}115}{12}\right)+12\ 000\left(1+\frac{0{,}115}{12}\right)$

$= R\ 45\ 312{,}83$ (to nearest cent). Hence another R4 687,17 will be needed.

4.1 $1{,}8(0{,}865)^8 = R\ 564\ 159$ (to nearest Rand)

4.2 $1{,}8(1{,}055)^8 = R\ 2\ 762\ 357$ (to nearest Rand)

5.1 $1+i = \left(1+\frac{0{,}12}{12}\right)^{12} \quad \therefore i = 12{,}68\%$ (corr. to 2 dec. pl.)

5.2 $1+i = \left(1+\frac{0{,}105}{365}\right)^{365} \quad \therefore i = 11{,}07\%$ (corr. to 2 dec. pl.)

5.3 $1+i = \left(1+\frac{0{,}13}{4}\right)^{4} \quad \therefore i = 13{,}65\%$ (corr. to 2 dec. pl.)

6.1 $1{,}0877 = \left(1+\frac{i}{4}\right)^4 \quad \therefore i = 4\left[\sqrt[4]{1{,}0877}-1\right] = 8{,}5\%$ (corr. to 1 dec. pl.)

6.2 $1{,}12 = \left(1+\frac{i}{12}\right)^{12} \quad \therefore i = 11{,}4\%$ (corr. to 1 dec. pl.)

6.3 $1{,}105 = \left(1+\frac{i}{365}\right)^{365} \quad \therefore i = 10{,}0\%$ (corr. to 1 dec. pl.)

7. $180\ 000 = x + x(1{,}01)^1 + x(1{,}01)^2 + ... + x(1{,}01)^{119}$

$= \frac{x\left[1{,}01^{120}-1\right]}{0{,}01}$

$\therefore x = \frac{180\ 000 \times 0{,}01}{1{,}01^{120}-1} = R\ 782{,}48$ (to nearest cent)

8. $250\ 000 = x\left(1+\frac{0{,}1}{12}\right) + x\left(1+\frac{0{,}1}{12}\right)^2 + x\left(1+\frac{0{,}1}{12}\right)^3 + ... + x\left(1+\frac{0{,}1}{12}\right)^{48}$

$\therefore x = \frac{250\ 000\left(\frac{0{,}1}{12}\right)}{\left(1+\frac{0{,}1}{12}\right)\left[\left(1+\frac{0{,}1}{12}\right)^{48}-1\right]} = R\ 4\ 222{,}13$ (to nearest cent)

9.1 $510\ 000 = x\left(1+\frac{0{,}14}{12}\right)^{-1} + x\left(1+\frac{0{,}14}{12}\right)^{-2} + x\left(1+\frac{0{,}14}{12}\right)^{-3} + ... + x\left(1+\frac{0{,}14}{12}\right)^{-240}$

$$\therefore 510\ 000 = \frac{x\left[1-\left(1+\frac{0{,}14}{12}\right)^{-240}\right]}{\frac{0{,}14}{12}}$$ (Using the formula $P = \frac{x\left[1-(1+i)^{n}\right]}{i}$)

Or using the sum formula for a geometric series:

$$510\ 000 = \frac{x\left(1+\frac{0{,}14}{12}\right)^{-1}\left[1-\left(1+\frac{0{,}14}{12}\right)^{-n}\right]}{\left[1-\left(1+\frac{0{,}14}{12}\right)^{-1}\right]}$$

In either case $x = R\ 6\ 341{,}96$ (to nearest cent)

9.2 $$510\ 000 = \frac{10\ 000\left(1+\frac{0{,}14}{12}\right)^{-1}\left[1-\left(1+\frac{0{,}14}{12}\right)^{-n}\right]}{\left[1-\left(1+\frac{0{,}14}{12}\right)^{-1}\right]}$$

$$\therefore n = \frac{\log\left[1-\frac{510\ 000\left(1-\left(1+\frac{0{,}14}{12}\right)^{-1}\right)}{10\ 000\left(1+\frac{0{,}14}{12}\right)^{-1}}\right]}{-\log\left(1+\frac{0{,}14}{12}\right)} = 78$$ months (to the nearest month)

9.3 It is perhaps surprising that by paying an extra R 3 658,04 each month the loan is paid off in only $6\frac{1}{2}$ years.

1.1 $\lim\limits_{x\to -2}\dfrac{(x+2)(x-2)}{x+2}=\lim\limits_{x\to -2}(x-2)=-4$

1.2 $\lim\limits_{h\to 0}\dfrac{x^2+2xh+h^2-x^2}{h}=\lim\limits_{h\to 0}\dfrac{h(2x+h)}{h}=2x$

2.1 $f'(x)=\lim\limits_{h\to 0}\dfrac{-\dfrac{2}{x+h}-\left(-\dfrac{2}{x}\right)}{h}=\lim\limits_{h\to 0}\dfrac{-2x+2h+2x}{hx(x+h)}=\lim\limits_{h\to 0}\dfrac{2}{x(x+h)}=\dfrac{2}{x^2}$

2.2 $g'(-1)=\lim\limits_{h\to 0}\dfrac{\left[2-(-1+h)^2\right]-\left[2-(-1)^2\right]}{h}=\lim\limits_{h\to 0}\dfrac{h(2-h)}{h}=2$

2.3 $D_x[6]=\lim\limits_{h\to 0}\dfrac{6-6}{h}=0$

3.1 $y=5x^{\frac{3}{2}}-3x^{\frac{1}{2}}\quad\therefore\dfrac{dy}{dx}=\dfrac{15}{2}x^{\frac{1}{2}}-\dfrac{3}{2}x^{-\frac{1}{2}}=\dfrac{15\sqrt{x}}{2}-\dfrac{3}{2\sqrt{x}}$

3.2 $D_x\left[x^3+2x^2-x-2\right]=3x^2+4x-1$

3.3 $f'(x)=x^3-2x^2+2x-1$

3.4 $\dfrac{d}{dx}\left(\dfrac{x^3}{3}-3x^{-3}\right)=x^2+\dfrac{9}{x^4}$

3.5 $y=2x^3-3x^2+2x-3\quad\therefore\dfrac{dy}{dx}=6x^2-6x+2$

3.6 $g(x)=2x^{\frac{3}{2}}-3x^{\frac{1}{2}}+2x^{-\frac{1}{2}}\quad\therefore g'(x)=3x^{\frac{1}{2}}-\dfrac{3}{2}x^{-\frac{1}{2}}-x^{-\frac{3}{2}}=3\sqrt{x}-\dfrac{3}{2\sqrt{x}}-\dfrac{1}{\sqrt{x^3}}$

4.1 When $t=2,\quad s=5(2)^2=20m$

4.2 Average speed = $\dfrac{45-20}{3-2}=25m/s$

4.3 $\dfrac{ds}{dt}=10t\quad\therefore$ instantaneous speed when $t=3$ is 30 $m/s$

4.4 Put $10t=20\quad\therefore t=2$

4.5 $5t^2=320\quad\therefore t=8$

4.6 Speed = $80m/s$

5.1 At A,B and C: $x^3-5x^2-8x+12=0$
Let $f(x)=(x-1)(x^2-4x-12)=(x-2)(x-6)(x+2)$
$A(-2;0),\quad B(1;0),\quad C(6;0)$

5.2 At D $f'(x)=3x^2-10x-8=0\ \therefore(3x+2)(x-4)=0$ and D is $(4;-36)$

5.3 $f''(x)=6x-10=0\ \therefore x=\dfrac{5}{3}$ and point of inflection is $(1{,}67;-10{,}59)$ (corr. to 2 dec. pl.)

5.4 At E $(0;12)$: $f'(0)=-8$ So tangent is $y=-8x+12$

5.5 At F: $f'(x)=3x^2-10x-8=-8\quad\therefore x(3x-10)=0$ and $x=\dfrac{10}{3}$

6.1 $AD = \frac{2\ 400}{x} m$

6.2 Length of fencing $l(x) = 2.\frac{2\ 400}{x} = 3x \quad l'(x) = -\frac{4\ 800}{x^2} + 3 = 0$

Minimum length of fencing when $x = 40m$

7.1 Let price increase = R 2 000 $x$ Then number sold = $40 - x$ and

weekly income = R $(40 - x)(72\ 000 + 2\ 000x)$ = $2\ 880\ 000 + 8\ 000x - 2\ 000x^2$

For maximum income: $8\ 000 - 4\ 000x = 0$ ie $x = 2$ and number of machines = 38

7.2 Weekly income = R 2 888 000

8. $V = \pi r^2 h \quad \therefore h = \frac{V}{\pi r^2}$

Surface area $A = 2\pi r^2 + 2\pi r h = 2\pi r^2 + 2\pi r.\frac{V}{\pi r^2} = 2\pi r^2 + \frac{2V}{r}$

For minimum surface area: $A' = 4\pi r - \frac{2V}{r^2} = 0 \quad \therefore r^3 = \frac{V}{2\pi}$

Since $h = \frac{V}{\pi r^2}$, $h^3 = \frac{V^3}{\pi^3 r^6} = \frac{V^3}{\pi^3} \times \frac{4\pi^2}{V^2} = \frac{4V}{\pi}$ (substituting for $r^6$)

$\therefore \frac{h^3}{r^3} = \frac{4V}{\pi} \times \frac{2\pi}{V} = 8$

So the ratio $h : r = 2 : 1$

## QUESTION 1

Note: *x* and *y* may be swopped in the definitions and the equation

*x* = yellow pigment; *y* = blue pigment; $x \geq 2y$

*x* = hours on Maths homework; *y* = hours on English homework; $x + y \leq 2$

*x* = first number; *y* = second number; $x + y \geq 12$

*x* = ice cream profit; *y* = chocolate profit; $x = 1{,}5y$

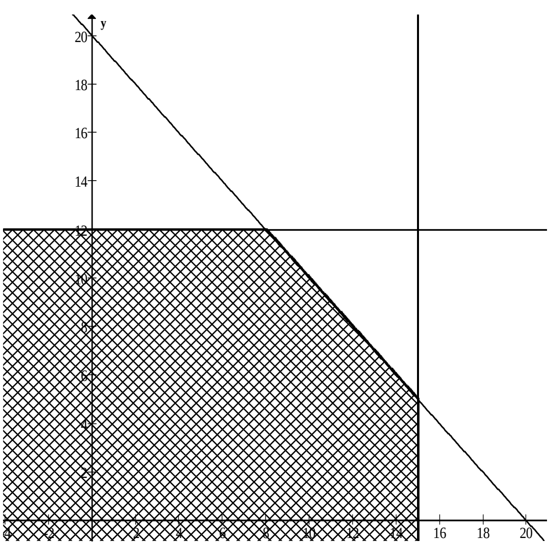
*x* = number of rooms with 2 people; *y* = number of rooms with three people; $3x + 3y < 250$

*x* = first number; *y* = second number; $M = x + y$

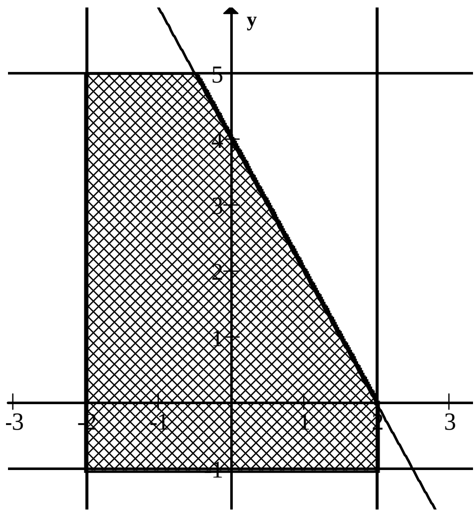
*x* = first number; *y* = second number; $M = x + 2y$

## QUESTION 2

2.1

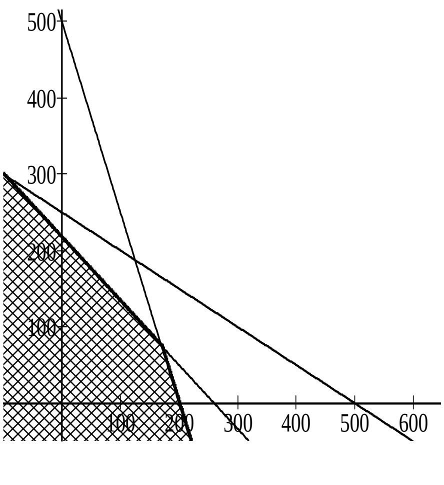


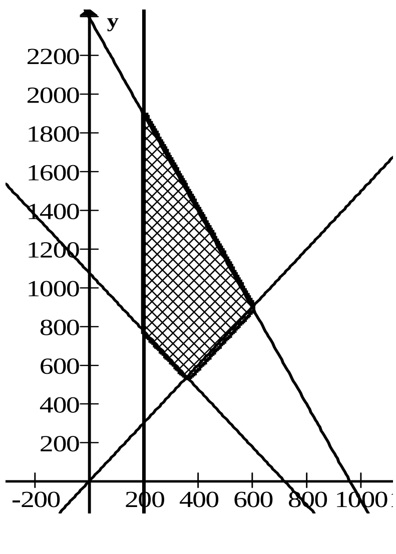

2.2




2.3

2.4



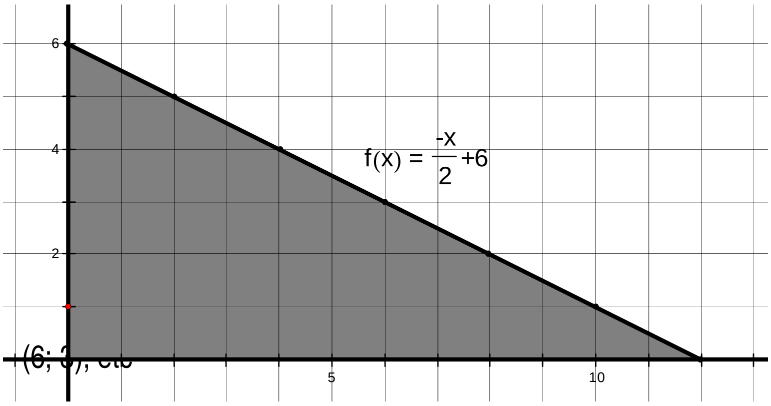



## QUESTION 3

3.1 $50 \leq x \leq 250$
$25 \leq y \leq 200$
$x + y \leq 300$

3.2 $50 \leq x \leq 200$
$y \geq 50$
$x + y \geq 150$
$5x + 6y \leq 1500$

## QUESTION 4

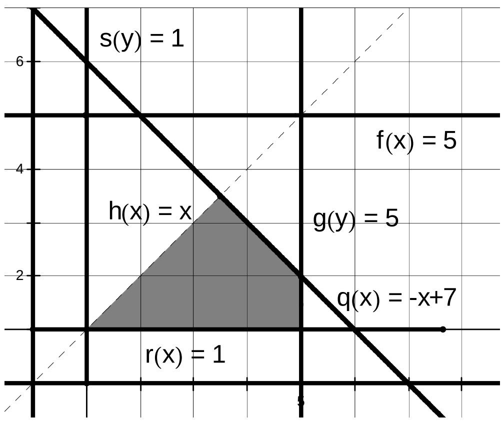



(2; 5) (4; 4) (6; 3), etc
½
On graph
(12; 0) (0; 6)
(1; 5)

## QUESTION 5

5.1 $x \leq 5$; $y \leq 5$; $x > y$; $x + y \leq 7$

5.2

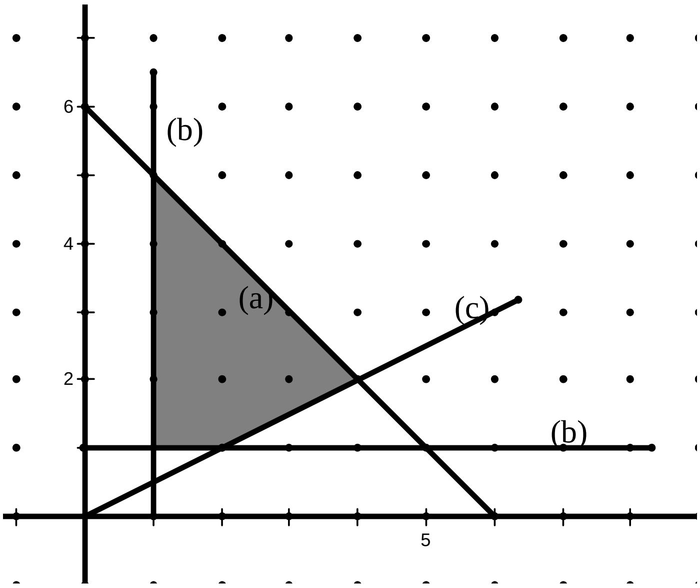



5.3 (2; 1) (3; 1) (3; 2) (4; 1) (4; 2) (4; 3) (5; 1) (5;2)

5.4 (4; 3) and (5; 2)

**QUESTION 6**

6.1




6.2 $x \geq 0$ *and* $y \geq 0$

6.3 On diagram

6.4 4

6.5 The added fruit may not exceed 6 scoops; $x + y \leq 6$

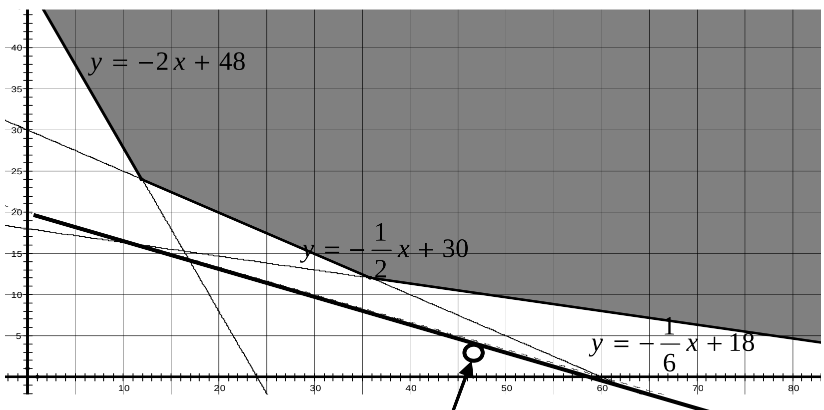
6.6 2 or 3

6.7 4 strawberry and 2 melon

**QUESTION 7**

7.1 $20x + 10y \geq 480$; $10x + 20y \geq 600$; $10x + 60y \geq 1080$; $x \geq 0$; $y \geq 0$

7.2




7.3 $C = 15x + 45y$

7.4 On graph

7.5
$$-\frac{1}{2}x + 30 = -\frac{1}{6}x + 18$$
$$-3x + 180 = -x + 108$$
$$2x = 72$$
$$x = 36$$
$$y = -18 + 30$$
$$y = 12$$

## QUESTION 8

8.1.1 $x \geq y$ ; area under lavender at least equal to area under vegetables.

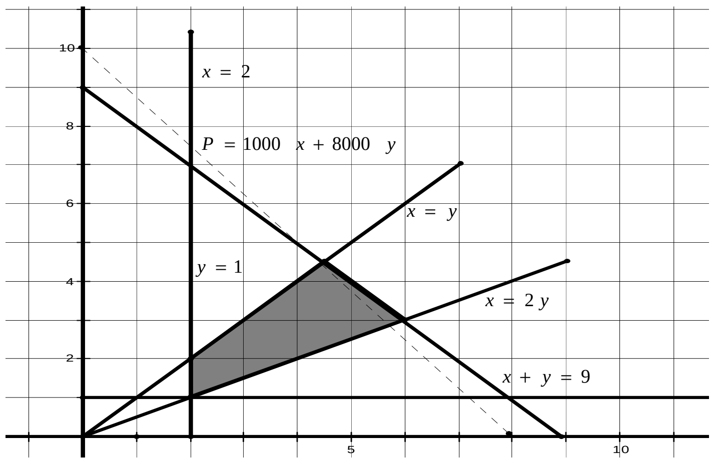
8.1.2 $x \geq 2$ ; plant at least two hectares of lavender

8.1.3 $x \leq 2y$ ; area under lavender not more than double vegetable area

8.1.4 $x + y \leq 9$ ; plot of land with area 9 hectares

8.1.5 $y \geq 1$; plant at least 1 hectare of vegetables

8.2




8.3 $P = 1000x + 8000y$

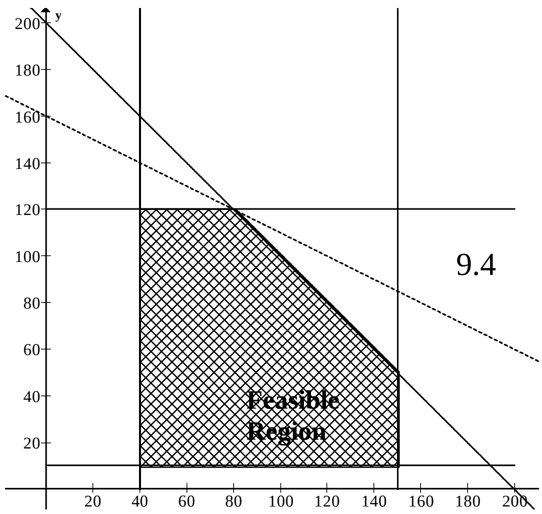
8.4 6 hectares of lavender and 3 hectares of vegetables.

8.5 The profit line would have the same gradient as $x + y \leq 9$. Any ordered pair on the portion of the line that lies in the feasible region would yield maximum profit. The current solution does lie on the line, therefore no change necessary, but a change could be made without affecting profitability.

## QUESTION 9

9.1 $40 \leq x \leq 150$
$10 \leq x \leq 120$
$x + y \leq 200$

9.2




9.3 P = $5x + 10y$

9.4 Max Profit = 5(80) + 10(120)
= R1600

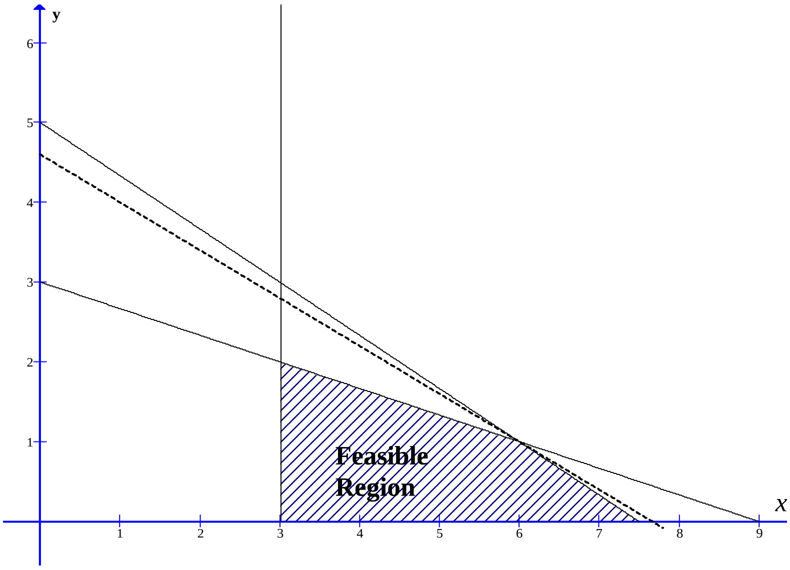
**QUESTION 10**

10.1 $3x + 9y \leq 27$

$4x + 6y \leq 30$

$x \geq 3$

10.2




10.3 P = 30$x$ + 50$y$

10.4 $x$ = 6 i.e. 6 litres of Energex and 1 litres of Vitagex

10.5 Max profit = 30(6) + 1(50) = R 230

1.1 $x^2 + y^2 = 3$

1.2 $x^2 + y^2 = 25$

1.3 $(x-2)^2 + (y-3)^2 = 36$

1.4 $(x-1)^2 + (y+7)^2 = 5$

1.5 $(x+3)^2 + (y+2)^2 = 45$

2.1 $\text{centre of circle} = \text{midpoint of PQ} = \left(\frac{12+(-12)}{2}; \frac{-5+5}{2}\right) = (0;0)$

2.2 $x^2 + y^2 = 13$

2.3 $(11)^2 + (6)^2 = 157 < 169$ $\therefore$ T lies inside the circle

3.1 $r^2 = (-4)^2 + (-1)^2 = 17$

$\therefore 17 = x^2 + y^2$

3.2 $m_{KO} = \frac{-1-0}{-4-0} = \frac{1}{4}$

$\therefore m_{KP} = -4$ $\quad\because\ m_{KO} \times m_{KP} = -1,\ KO \perp KP$

$(y - y_K) = m(x - x_K)$

$(y+1) = -4(x+4)$

$y + 1 = -4x - 16$

$y = -4x - 17$

3.3 P is the *x*-intercept $\therefore$ P$\left(-\frac{17}{4}; 0\right)$

Q is the *y*-intercept $\therefore$ Q(0 ; – 17)

4.1 M(0 ; -3)

4.2 Not a tangent: putting $x = y - 1$ in the equation of the circle, two distinct roots are obtained.

5.1 $(x+1)^2+(y+5)^2=25$

5.2 $m_{MR}=\frac{-5-(-1)}{-1-(-4)}=\frac{-4}{3}=-\frac{4}{3}$

$\therefore m_{\tan gent}=-\frac{1}{2}$

$y+1=\frac{3}{4}(x+4)$

$4y+4=3x+12$

$4y=3x+8$

6.1 $y$-coordinate of C = 2
$\therefore$ $3x + 4(2) + 7 = 0$
$\therefore$ $x = -5$
$(x+5)^2+(y-2)^2=25$

6.2 DE = 10

6.3 $4x - 3y - 51 = 0$

7.1 M(– 2 ; 6) ; $r = 6$

7.2 (– 2 ; 0)

7.3 $x = -8$ and $x = 4$

8.1 $y = -x + 8$

8.2 P(3 ; 5)

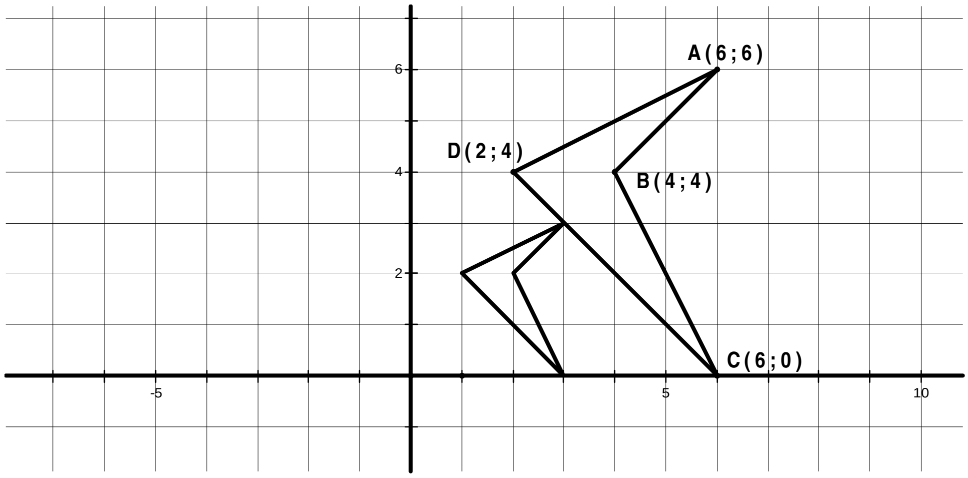
8.3 $(x-4)^2+(y-4)^2=2$

8.4 T(5 ; 3)

8.5 $2\sqrt{2}$

### Grade 12 Transformation Geometry

1.1.1




1.1.2 $A'(3;3)$ $C'(3;0)$

1.1.3 $Area = \left(\frac{1}{2}\right)^2 (2x) = \frac{x}{2}$

1.2.1 $(x;y) \rightarrow (y'-x)$

1.2.2 $A''(6;-6)$ $B''(4;-4)$ $C''(0;-6)$ $D''(4;-2)$

2.1 reflection about the x-axis

2.2 $(x;y) \rightarrow (x;-y)$

2.3 $P'(2;-1)$

2.4 translation 4 up and 6 to the right

2.5 $(x;y) \rightarrow (x+6;y+4)$

2.6 $M(-5;2) \rightarrow (-5;-2) \rightarrow M'(-2;-5)$

2.7 $H(3;-3) \rightarrow (3;3) \rightarrow H'(-3;3)$

3.1 reflection about the line $y = x$

3.2 $(x;y) \rightarrow (y;x)$

3.3 a) reflection about both the x and y axis (in any order)

b) rotation through $180^\circ$ about the origin

3.4 B

3.5 reflection in the x-axis followed by translation 3 left and 2 down (any order)

3.6 rotation through $90^\circ$ anti-clockwise about the origin

3.7 $(x;y) \rightarrow (-y;x)$

4.1 $S(-1;4)$

$x' = (-1)\cos 120^\circ - 4\sin 120^\circ$

$\therefore x' = \cos 60^\circ - 4\sin 60$

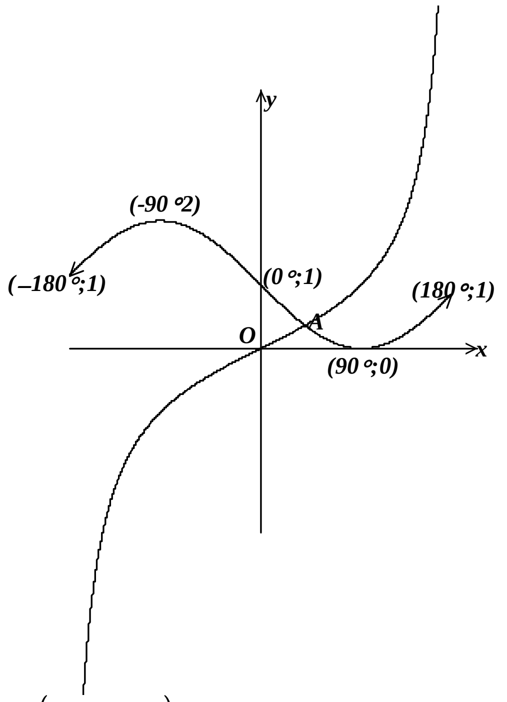
$\therefore x' = -2{,}96$

$y' = 4\cos 120^\circ + (-1)\sin 120^\circ$

$\therefore y' = -4\cos 60^\circ - \sin 60^\circ$

$\therefore y' = -2{,}86$

1.1



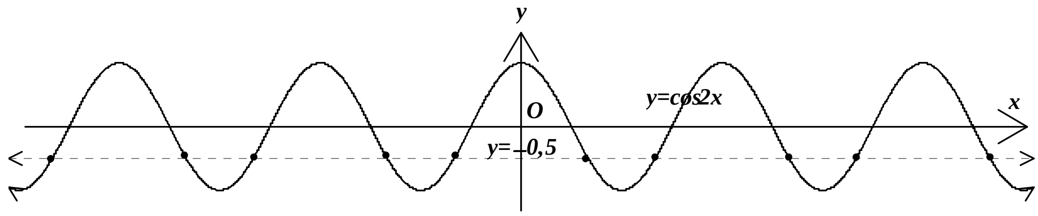

1.2 $x \in \left(40^0;180^0\right)$ (Read from A to $180^0$)

2.1 $2\cos 2x = -1$

$\therefore \cos 2x = -\frac{1}{2}$

$\therefore 2x = \pm 120^0 + k.360^0;\ k \in Z$

$\therefore x = \pm 60^0 + k.180^0;\ k \in K$




Solutions at points of intersection of $y = \cos 2x$ and $y = -0,5$

2.2 $\sin x = 3\cos x$

$\therefore \tan x = 3$

$\therefore x = \tan^{-1}(3) + k.180^0;\ k \in Z = 71,47^0 + k.180^0;\ k \in Z$

2.3 $\sin x = \cos 3x$

$\therefore \cos\left(90^0 - x\right) = \cos 3x$

$\therefore 90^0 - x = \pm 3x + k.360^0$

$\therefore 4x = 90^0 + k.360^0$ or $2x = -90^0 + k.360^0$

$\therefore x = 22,5^0 + k.90^0$ or $x = -45^0 + k.180^0 \quad k \in Z$

2.4 $6 - 10\cos x - 3\left(1 - \cos^2 x\right) = 0$

$\therefore 3\cos^2 x - 10\cos x + 3 = 0$

$\therefore (3\cos x - 1)(\cos x - 3) = 0$

$\therefore \cos x = \frac{1}{3}$ or $\cos x = 3$

$\therefore x = \pm\cos^{-1}\left(\frac{1}{3}\right) + k.360^0$ invalid

$\therefore x = \pm 70{,}53^0 + k.360^0;\ k \in Z$

For $x \in [-360^0;360^0]$ $x \in \{-289{,}47^0;-70{,}53^0;70{,}53^0;289{,}47^0\}$ (corr. to 2 dec pl.)

2.5 $2(\sin^2 x + \cos^2 x) - \sin x\cos x - 3\cos^2 x = 0$

$\therefore 2\sin^2 x - \sin x\cos x - \cos^2 x = 0$

$\therefore (2\sin x + \cos x)(\sin x - \cos x) = 0$

$\therefore \tan x = -0{,}5$ or $\tan x = 1$

$\therefore x = \tan^{-1}(-0{,}5) = k.180^0$ or $x = 45^0 + k.180^0;\ k \in Z$

$\therefore x = -26{,}57^0 + k.180^0;\ k \in Z$

2.6 $3(\sin^2 x + \cos^2 x) - 8\sin x + 16\sin x\cos x - 6\cos x = 0$

$\therefore 3 - 6\cos x - 8\sin x + 16\sin x\cos x = 0$

$\therefore 3(1 - 2\cos x) - 8\sin x(1 - 2\cos x) = 0$

$\therefore (1 - 2\cos x)(3 - 8\sin x) = 0$

$\therefore x = \pm 60^0 + k.360^0; k \in Z$ or $\sin x = 0{,}375$

$\therefore x = 22{,}02^0$ or $x = 157{,}98^0$

$+ k.360^0; k \in Z$

3.1 LHS= $\cos x + \frac{\sin x}{\cos x} \times \sin x$

$= \frac{\cos^2 x + \sin^2 x}{\cos x}$

$= \frac{1}{\cos x}$

=RHS Not valid for $x = 90^0 + k.180^0;\ k \in Z$

3.2 LHS = $\frac{\sin^2\theta - \cos\theta(1-\cos\theta)}{(1-\cos\theta)\sin\theta} = \frac{\sin^2\theta + \cos^2\theta - \cos\theta}{(1-\cos\theta)\sin\theta} = \frac{1-\cos\theta}{(1-\cos\theta)\sin\theta} = \frac{1}{\sin\theta}$

=RHS

Not valid for $\theta = k.180^0;\ k \in Z$

3.3 LHS = $\frac{1-\cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x} = \frac{\sin x}{\cos x}.\sin x = \tan x.\sin x$ = RHS

Not valid for $x = 90^0 + k.180^0;\ k \in Z$

3.4 LHS = $\frac{\sin x(\sin^2 x + \cos^2 x)}{\cos x} = \frac{\sin x}{\cos x} = \tan x$ =RHS

Not valid for $x = 90^0 + k.180^0;\ k \in Z$

3.5 $LHS = \frac{1 + \frac{\sin x}{\cos x}}{1 - \frac{\sin x}{\cos x}} = \frac{\cos x + \sin x}{\cos x} \times \frac{\cos x}{\cos x - \sin x} \times \frac{\cos x + \sin x}{\cos x + \sin x}$

$= \frac{\cos^2 x + 2\sin x\cos x + \sin^2 x}{\cos^2 x - \sin^2 x} = \frac{1 + 2\sin x\cos x}{\cos^2 x - \sin^2 x} = RHS$

Not valid for $x = \pm 45^0 + k.180^0;\ k \in Z$

4.1 $\dfrac{\sin(180^0 - x)\tan(-x)}{\tan(180^0 + x)\cos(x - 90^0)} = \dfrac{\sin x(-\tan x)}{\tan x(\sin x)} = -1$

4.2 $\dfrac{\sin(180^0 + x)\tan(x - 360^0)}{\tan(360^0 - x)(-\cos 60^0)(\tan 45^0)} = \dfrac{\sin x.\tan x}{-\tan x(-0{,}5)(1)} = 2\sin x$

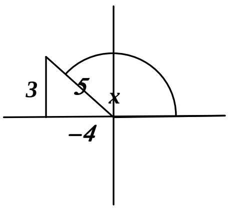
5.1 $\cos 73^0 = \cos(90^0 - 17^0) = \sin 17^0 = a$

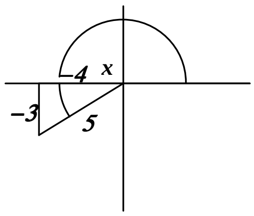
5.2 $\cos -163^0 = \cos 163^0 = \cos(180^0 - 17^0) = -\cos 17^0 = -\sqrt{1 - a^2}$

5.3 $\tan 197^0 = \tan(180^0 + 17^0) = \dfrac{\sin 17^0}{\cos 17^0} = \dfrac{a}{\sqrt{1 - a^2}}$

5.4 $\cos 326^0 = \cos 34^0 = \cos 2 \times 17^0 = 1 - 2\sin^2 17^0 = 1 - 2a^2$

6.



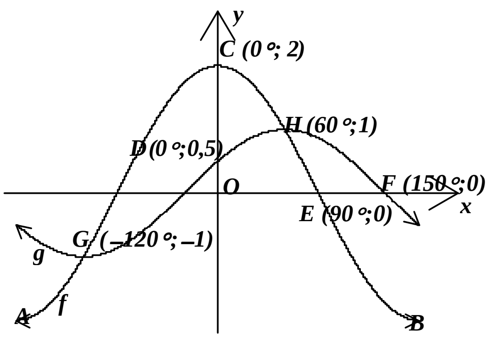

6.1 $5\sin x + 3\tan x = 5\left(\dfrac{3}{5}\right) + 3\left(\dfrac{3}{-4}\right)$ or $= 5\left(\dfrac{-3}{5}\right) + 3\left(\dfrac{-3}{-4}\right)$

$= 3 - \dfrac{9}{4} = \dfrac{3}{4}$ or $= -5 + \dfrac{9}{4} = -\dfrac{3}{4}$

6.2 $\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

$\therefore \tan 2x = \dfrac{2\left(\dfrac{3}{-4}\right)}{1 - \left(\dfrac{3}{-4}\right)^2} = -\dfrac{3}{2} \times \dfrac{16}{6} = -\dfrac{24}{7}$ or $\tan 2x = \dfrac{3}{2} \times \dfrac{16}{7} = \dfrac{24}{7}$

7.1




7.2.1 Period of $f$ is $360^0$ from A$(-180^0;-2)$ to B $(180^0;-2)$

7.2.2 $f(0) - g(0) = 2 - 0{,}5 = 1{,}5$ (Read from C to D)

7.2.3 $2\sin(x + 30^0)\cos x < 0$ when one graph is above and the other below the $x$-axis

ie. from E to F $90^0 < x < 150^0$

7.2.4 $2\cos x = \sin(x + 30^0)$

$\therefore 2\cos x = \sin x \cos 30^0 + \cos x \sin 30^0$

$\therefore 2\cos x = \sin x.\frac{\sqrt{3}}{2} + \cos x\frac{1}{2}$

$\therefore \frac{3}{2}\cos x = \frac{\sqrt{3}}{2}\sin x$

$\therefore \frac{3}{2}\times\frac{2}{\sqrt{3}} = \frac{\sin x}{\cos x}$

$\therefore \tan x = \sqrt{3}$

$\therefore x = 60^0 + k.180^0; k \in Z$

Hence G is the point $(-120^0;-1)$ and H is the point $(60^0;1)$

7.2.5 The equation would be $y = 2\cos x - 1$

8.1 $a = -2, \quad b = 60^0$

8.2 $C(0^0;-1), \quad D(30^0;0)$

9.1 $\beta = 180^0 - 47^0 = 133^0$

9.2 In $\Delta ABC$ $\frac{150}{AC} = \cos 35^0 \quad \therefore AC = \frac{150}{\cos 35^0} = 183m$ (corr. to nearest $m$)

9.3 In $\Delta ACD$ $AD^2 = 300^2 + AC^2 - 2(300)AC\cos 133^0$

By the Sine Rule $\sin\alpha = \frac{300\sin 133^0}{AD} \quad \therefore \alpha = 29{,}51^0$ (corr. to 2 dec. pl.)

9.4 $\alpha + \theta = 35^0 \quad \therefore \theta = 5{,}49^0$ (corr. to 2 dec. pl.)

10.1 $\cos B\hat{D}A = \frac{7^2 + 8^2 - 13^2}{2.7.8} = -\frac{1}{2} \quad \therefore B\hat{D}A = 120^0$

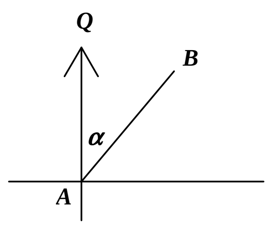
10.2 $AC^2 = 8^2 + 5^2 - 2.8.5.\cos 60^0 \quad \therefore AC = 7$

10.3 Area = $\frac{1}{2}7.8.\sin 120^0 + \frac{1}{2}8.5.\sin 60^0 = 4\sin 60^0(7+5) = 4\times\frac{\sqrt{3}}{2}\times 12 = 24\sqrt{3}\, m^2$

11. $AD = 13$ (Th. of Pythagoras) In $\Delta ACD$ $\frac{CD}{\sin[180^0 - (\alpha+\beta)]} = \frac{13}{\sin\alpha}$

$$\therefore CD = \frac{13\sin(\alpha+\beta)}{\sin\alpha}$$

12.




In $\Delta PQB$ $\frac{x}{QB} = \tan\theta \quad \therefore QB = \frac{x}{\tan\theta}$

Similarly $AQ = \frac{x}{\tan\theta}$ and hence $Q\hat{B}A = \alpha$

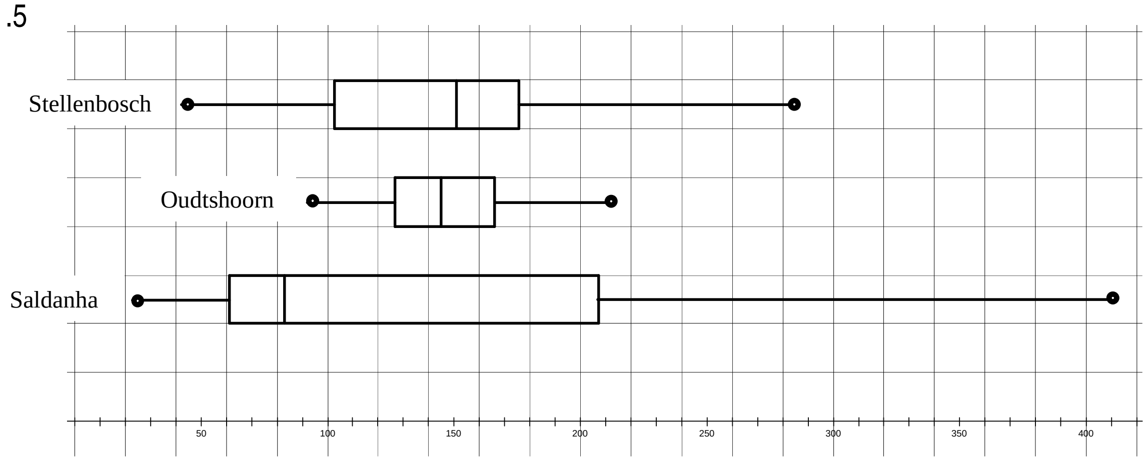
$\Delta QAB$ $\frac{AB}{\sin(180^0 - 2\alpha)} = \frac{QB}{\sin\alpha} \quad \therefore AB = \frac{QB\sin 2\alpha}{\sin\alpha} = \frac{x}{\tan\theta}\times\frac{2\sin\alpha\cos\alpha}{\sin\alpha}$

$\therefore AB = \frac{2x\cos\alpha}{\tan\theta}$

**Question 1**

| | | Saldanha | Oudtshoorn | Stellenbosch |
|---|---|---|---|---|
| | | R 52.00 | R 103.00 | R 183.00 |
| | | R 112.00 | R 92.00 | R 104.00 |
| | | R 83.00 | R 168.00 | R 226.00 |
| | | R 256.00 | R 140.00 | R 101.00 |
| | | R 412.00 | R 146.00 | R 92.00 |
| | | R 61.00 | R 183.00 | R 286.00 |
| | | R 54.00 | R 214.00 | R 63.00 |
| | | R 81.00 | R 177.00 | R 42.00 |
| | | R 382.00 | R 145.00 | R 235.00 |
| | | R 134.00 | R 135.00 | R 152.00 |
| | | R 61.00 | R 164.00 | R 112.00 |
| | | R 78.00 | R 107.00 | R 151.00 |
| | | R 225.00 | R 139.00 | R 151.00 |
| | | R 23.00 | R 152.00 | R 168.00 |
| | | R 189.00 | R 118.00 | R 131.00 |
| 1.1 | 1.1.1 Mean | R 146.87 | R 145.53 | R 146.47 |
| | 1.1.2 Median | R 83.00 | R 145.00 | R 151.00 |
| 1.2 | The mean values for the three sets of data are very similar. The medians are substantially different, with Saldanha having a median that is R62 lower than that of Oudtshoorn and R68 lower than Stellenbosch. | | | |
| 1.3 | The data sets are small and any particular value is unlikely to occur even twice within the data set. The data does not have a mode. | | | |
| 1.4 | 1.4.1 Maximum | R 412.00 | R 214.00 | R 286.00 |
| | 1.4.2.Minimum | R 23.00 | R 92.00 | R 42.00 |
| | 1.4.3 Range | R 389.00 | R 122.00 | R 244.00 |
| | 1.4.4 Upper Quartile | R 207.00 | R 166.00 | R 175.50 |
| | 1.4.5 Lower Quartile | R 61.00 | R 126.50 | R 102.50 |
| | 1.4.6 Interquartile range | R 146.00 | R 39.50 | R 73.00 |

1.5

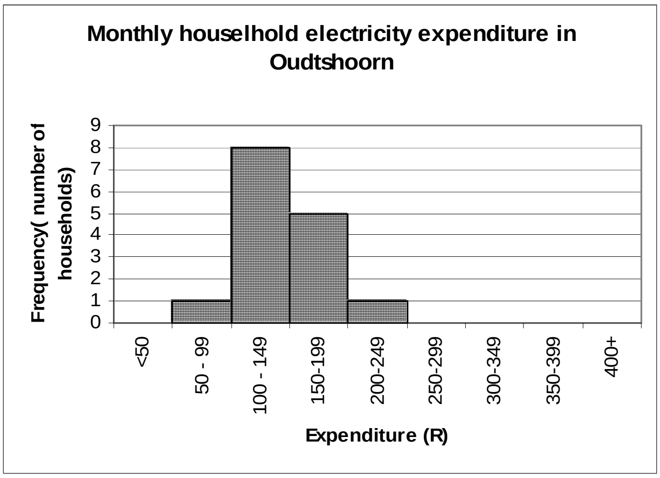


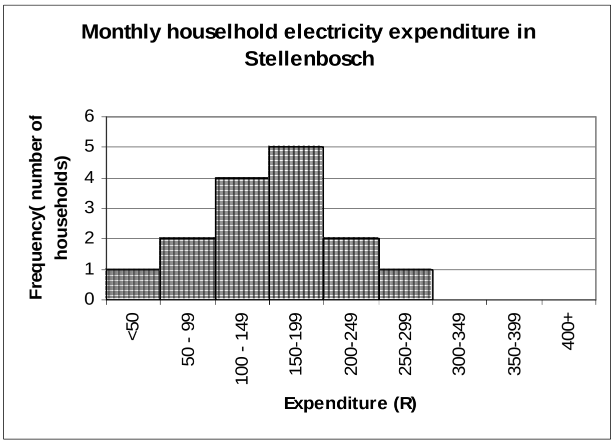
1.6 The mean household expenditure in all three municipalities is very similar and is approximately R146 per month. There is the least variability in expenditure in Oudtshoorn, with monthly electricity expenditure ranging from R92 to R214. This contrast with Saldanha, where the expenditure ranges form R23 to R412 and where half the residents spend less than R146 per month. Expenditure in Stellenbosch is more variable than Oudtshoorn, but less so than Saldanha. In Stellenbosch, expenditure ranges from R92 to R214, with half the residents spending between R126 and R214 per month on electricity.

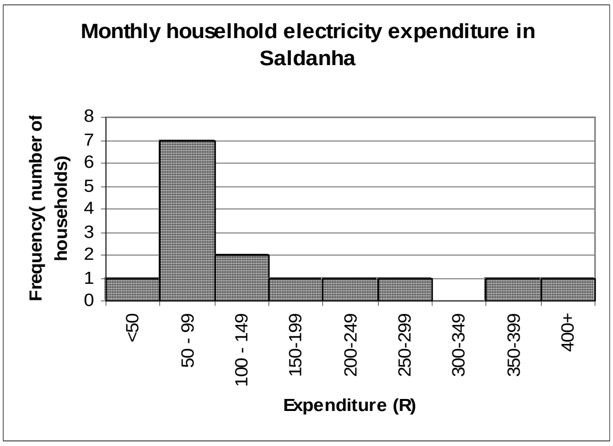
1.7 .1

| | Modal class | Median | Mean |
|---|---|---|---|
| Saldanha | R50 – R99 | R83 | R146.87 |
| Oudtshoorn | R100 – R149 | R145 | R145.53 |
| Stellenbosch | R150 - R199 | R151 | R146.47 |

1.7.2 The grouping of data allows for better summary of data items that are similar, but not exactly the same. Small individual differences in individual scores are negated and it is possible to establish whether or not expenditure falls more heavily into one or more classes of expenditure (modal class).



**Question 2**

2.1 Range = 134 minutes

2.2 Modal class: 75 – 89 minutes

2.3 Median:

$$\frac{2135}{2} = 1067,5$$

Class containing 1068th value is 90-104 minutes.

Median time taken to get to work is approximately 97,7 minutes.

| Class | Frequency | Cumulative Frequency |
|---|---|---|
| 0 - 14 minutes | 11 | 11 |
| 15 - 29 minutes | 193 | 204 |
| 30 - 44 minutes | 302 | 506 |
| 45 - 59 minutes | 254 | 760 |
| 60 - 74 minutes | 359 | 1119 |
| 75 - 89 minutes | 417 | 1536 |
| 90 - 104minutes | 287 | 1823 |
| 105 - 119 minutes | 105 | 1928 |
| 120 - 134 minutes | 207 | 2135 |

2.4

| Time taken to travel to work | Lower limit of class | Midpoint of class $X$ | Number of people $f$ | Class total $fX$ |
|---|---|---|---|---|
| 0 - 14 minutes | 0 | $\left(\frac{0+15}{2}\right)$ 7.5 | 11 | $(11 \times 7.5)$ 82.5 |
| 15 - 29 minutes | 15 | 22.5 | 193 | 4342.5 |
| 30 - 44 minutes | 30 | 37.5 | 302 | 11325 |
| 45 - 59 minutes | 45 | 52.5 | 254 | 13335 |
| 60 - 74 minutes | 60 | 67.5 | 359 | 24232.5 |
| 75 - 89 minutes | 75 | 82.5 | 417 | 34402.5 |
| 90 - 104minutes | 90 | 97.5 | 287 | 27982.5 |
| 105 - 119 minutes | 105 | 112.5 | 105 | 11812.5 |
| 120 - 134 minutes | 120 | 127.5 | 207 | 26392.5 |
| | | | $n$=2135 | $\left(\frac{\sum fX}{n}\right)$ 72 |

Estimated mean = 72 minutes

**Question 3**

3.1 $\text{Mean} = \dfrac{120310,00}{15}$

$= \text{R}8020,67$

3.2

| Oudsthoorn Income | $(x-\bar{x})$ | $(x-\bar{x})^2$ |
|---|---|---|
| R 5,534.00 | -R 2,486.67 | 6,183,511.11 |
| R 5,886.00 | -R 2,134.67 | 4,556,801.78 |
| R 6,231.00 | -R 1,789.67 | 3,202,906.78 |
| R 6,671.00 | -R 1,349.67 | 1,821,600.11 |
| R 7,004.00 | -R 1,016.67 | 1,033,611.11 |
| R 7,421.00 | -R 599.67 | 359,600.11 |
| R 7,821.00 | -R 199.67 | 39,866.78 |
| R 7,974.00 | -R 46.67 | 2,177.78 |
| R 8,023.00 | R 2.33 | 5.44 |
| R 8,368.00 | R 347.33 | 120,640.44 |
| R 8,541.00 | R 520.33 | 270,746.78 |
| R 8,718.00 | R 697.33 | 486,273.78 |
| R 9,687.00 | R 1,666.33 | 2,776,666.78 |
| R 10,355.00 | R 2,334.33 | 5,449,112.11 |
| R 12,076.00 | R 4,055.33 | 16,445,728.44 |
| | | 42,749,249.33 |

Standard Deviation = R1688.18

3.3 Interval = [8020,67-1688,18; 8020,67+1688,18]
= [6332,49; 9078,85]
10 households fall in this interval
Percentage = 66,67%

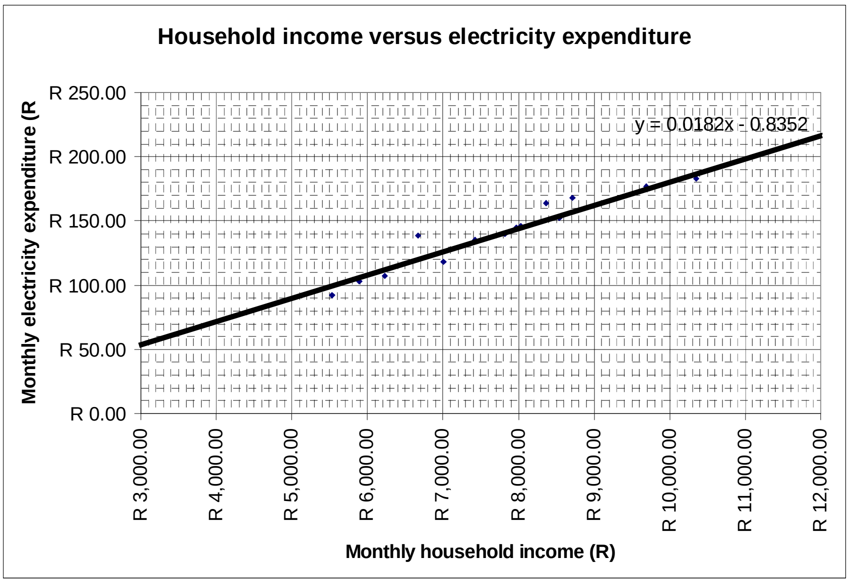
3.3.1 Graph on next page

On graph

3.3.2 Linear. The scatterplot clusters in a straight line – the rate of increase of expenditure in relation to income is more or less constant.

3.3.3 R55

3.3.4 The gradient will decrease and the graph will shift downwards.

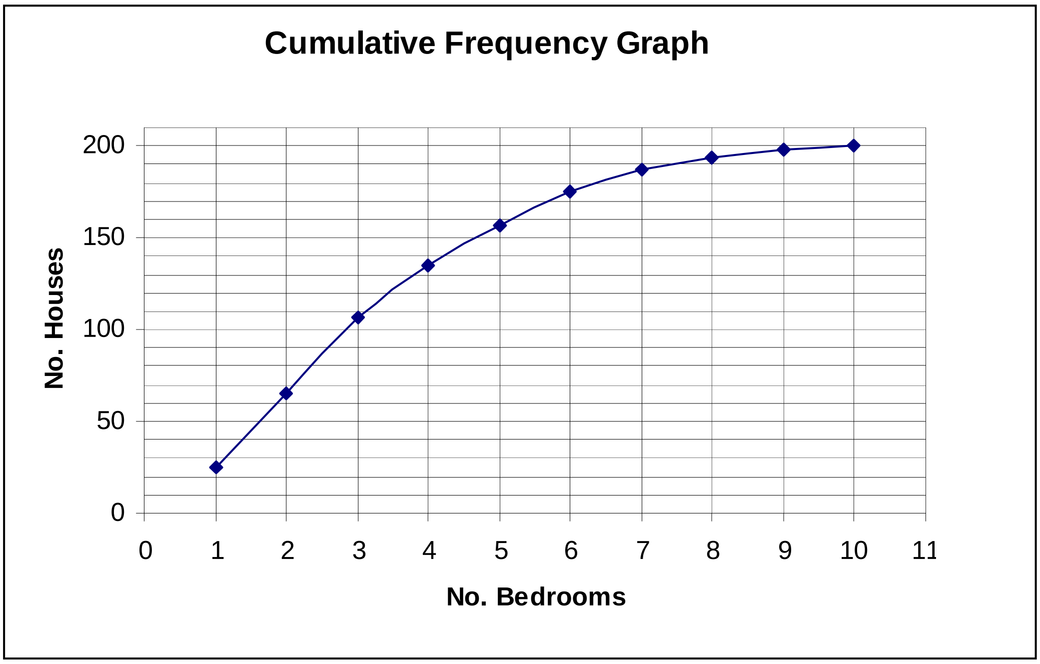



## Question 4

4.1

| No. Bedrooms | Frequency | Cumulative Frequency |
|---|---|---|
| 1 | 25 | 25 |
| 2 | 40 | 65 |
| 3 | 42 | 107 |
| 4 | 28 | 135 |
| 5 | 22 | 157 |
| 6 | 18 | 175 |
| 7 | 12 | 187 |
| 8 | 7 | 194 |
| 9 | 4 | 198 |
| 10 | 2 | 200 |

4.2 ***Ogive should join the x axis at (0;0)***



**Question 5**

5.1 16 %

5.2 84 %

5.3 2 %