Grade 12
Assessment Exemplars
2008

Grade 12 Assessment Exemplars

Information Sheet: Mathematics

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + i.n)$ $\quad A = P(1+i)^n$

$A = P(1-i)^n$ $\quad A = P(1 - i.n)$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad \sum_{i=1}^{n} (a + (i-1)d) = \frac{n}{2}(2a + (n-1)d)$

$\sum_{i=1}^{n} ar^{i-1} = \frac{a(r^n - 1)}{r - 1} \ ; \ r \neq 1$ $\quad \sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{r-1} \ ; \ -1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1 - (1+i)^{-n}]}{i}$ $\quad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$ $\quad y = mx + c$

$y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$ $\quad (x-a)^2 + (y-b)^2 = r^2$

$In\ \Delta ABC ;$ $\quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$

$\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$

$\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ $\quad \cos 2\alpha = 1 - 2\sin^2\alpha$ $\quad \cos 2\alpha = 2\cos^2\alpha - 1$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \bar{x} = \frac{\sum fx}{n}$ $\quad \text{var} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$SD = \sqrt{\frac{\sum_{i=1}^{n}(x - \bar{x})^2}{n}}$ $\quad P(A) = \frac{n(A)}{n(s)}$ $\quad P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

Instructions and Information

Read the following instructions carefully before answering this question paper:

1 This question paper consists of …… questions. Answer ALL questions.

2 Clearly show ALL calculations, diagrams, graphs, et cetera, which you have used in determining the answers.

3 An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

4 If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

5 Number your answers correctly according to the numbering system used in this question paper.

6 Diagrams are not necessarily drawn to scale.

7 It is in your own interest to write legibly and to present your work neatly.

Grade 12 Assignment: Functions

Given the following:

a) $f(x) = 2x^2 - 3x - 2$
b) $g(x) = -5x + 10$
c) $h(x) = x^3 - 12x - 16 = (x+2)^2(x-4)$

1 Explain why each of the above is a function.

2 $f(x)$, $g(x)$ and $h(x)$ are all polynomial functions. Explain what this means and give the degree of each function.

3 Explain what is meant by the zeros of the graphs.

4 Explain why the three functions do not cut the *x*-axis at the same number of points.

5 Why do all cut the *y*-axis at one point only? Explain.

6 Will the graphs have a minimum or a maximum value? Explain in each case. How can you determine possible minimum or maximum values.

7
- 7.1 On different systems of axes, draw neat sketch graphs of the above functions. Show all calculations.
- 7.2 On the same system of axes draw the inverse of each function.
- 7.3 In each case discuss whether the inverse is a function or not.
- 7.4 Explain how you would restrict the domain of any many-to-one functions above so that the inverse becomes a function.

8 What is the difference between $h(2)$ and $h'(2)$ and also between g(2) and $g'(2)$.

9 What is the value of the gradient of $h(x)$ at the maximum or minimum points?

10 On the same system of axes draw in different colours the graphs of $h(x)$ and $h'(x)$.

11 Which of the functions have points of inflection? Explain.

12 If $k(x)$ is any function, what is the relationship between the graphs of:

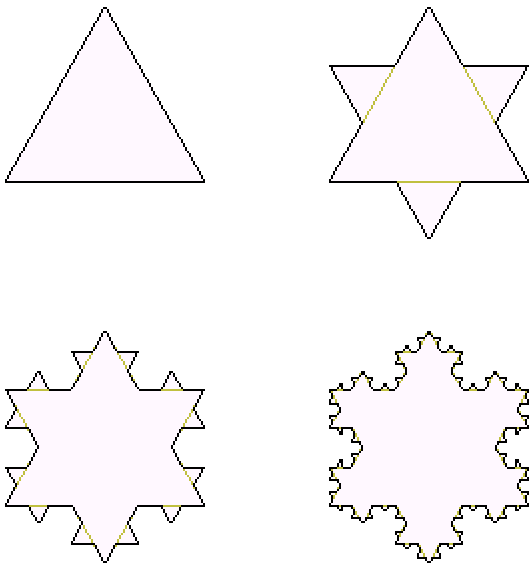
- 12.1 $k(x)$ and $-k(x)$
- 12.2 $k(x)$ and $k(-x)$
- 12.3 $k(x)$ and $k^{-1}(x)$

Grade 12 Investigation: The Koch Snowflake **Marks: 100**

The Mathematics described in this investigation was produced in 1904 by a Swedish mathematician, Helge Von Koch.

The snowflake begins with an equilateral triangle having sides 1 unit in length and hence a perimeter of 3 units.

Below are the first four Snowflake iterations.



Section A (to be done on triangular dotty paper)

Create Snowflakes 1, 2, 3 and 4 as follows:

Snowflake 1:
Draw an equilateral triangle having sides 1 unit in length and perimeter of 3 units.
(Hint: Let 1 unit = 9cm)

Snowflake 2:
Divide each side of the equilateral triangle into thirds, then draw another equilateral triangle on the middle portion by joining the thirds marked off on the original triangle.

Snowflake 3:
Repeat what was done for snowflake 2 for each new side of the diagram.

Snowflake 4:
Repeat what was done for snowflake 3

Section B

Copy and complete the table below with respect to the first 6 iterations of the Koch Snowflake. Clearly show all calculations

Snowflake	A: Length of 1 side	B: Number of sides	C: Perimeter
1	1	3	3
2		12	
3			
4			
5			
n			

2. The table shows that snowflake construction produces 3 separate sequences. Use the information calculated in the table to determine the n-th term and the type of sequence for A for B and for C.

3. Make at least 1 observation about *each* sequence.

4. As the lengths of the sides are getting smaller and smaller, what happens to the perimeter? Explain.

Section C

Investigate the increase in area of the Koch Snowflake at successive stages.

1 Call the area of the original triangle 1 unit, and complete the table below.

Snowflake	A: Area of each added Triangle	B: Number of Triangles added	C: Increase in Area	D: Total Area
1	1			1
2	1/9	3	3.(1/9)	1 + 3.(1/9)
3				
4				
5				
n				

2 Write down the sum of n-terms of the sequence in column C

3 What happens to this sum as $n \to \infty$?

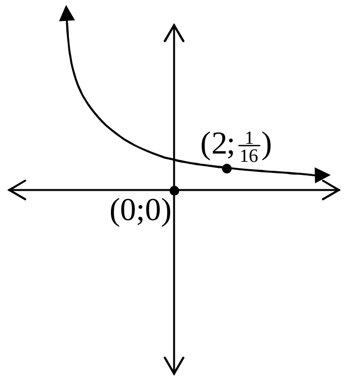
4 The difference between the perimeter and the area of the Koch Snowflake as $n \to \infty$ is very interesting. Comment on this difference.

Grade 12 Test: Number Patterns, Finance and Functions

Time: 1 hour **Marks: 50**

Question 1

1.1 The figure represents the graph of $f(x) = a^x$. Calculate the value of *a*. (2)



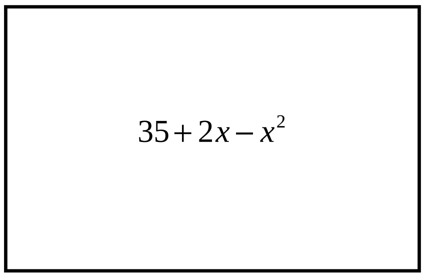
1.2 Draw a sketch graph of $k(x)$ if *k* is the inverse of *f*. Indicate the intercept(s), the coordinates of one other point and the asymptote(s). (3)

1.3 The function $h(x) = \dfrac{4}{x+p} + q$ has asymptotes at $x = 2$ and $y = -1$. Determine the values of *p* and *q.* (2)

[7]

Question 2

The area of the rectangle in the figure is given by the formula: $A = 35 + 2x - x^2$ where *A* is the area in square units.



2.1 What is meant by the statement *"the area of the rectangle is a function of x"*? Explain fully. (3)

2.2 For which value(s) of *x* will the rectangle have an area of 20 square units? (3)

2.3 Draw a sketch graph of $A = 35 + 2x - x^2$. Label the following clearly: intercept(s) with the axes, coordinates of turning point(s), equation(s) of axes of symmetry. (5)

2.4 Use the graph to answer the following questions:

2.4.1 What are the possible values of *x*? (2)
2.4.2 For which values of *x* will the area of the rectangle be decreasing? (2)
2.4.3 What is the maximum area of the rectangle? (2)

[17]

Question 3

Consider the following sequence: 6; 18; 54; 162 ……..

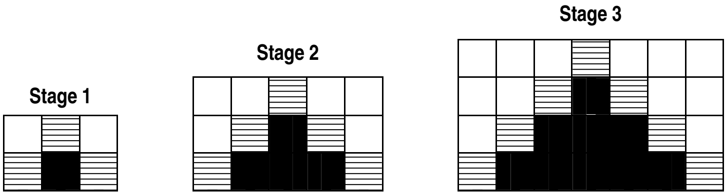
3.1 If the formula for the general term of the sequence is $T_n = ar^{n-1}$, give the value of *a* and *r*. (2)

3.2 Which term of the sequence is equal to 1458? (4)

[6]

Question 4

Examine the tiling pattern below:



4.1 Complete the table by working out Stage *n* for the number of patterned tiles, the number of black tiles and the total number of tiles. Show all your work. (5)

	Stage 1	Stage 2	Stage 3	Stage 4	Stage *n*
Number of patterned tiles	3	5	7	9	
Number of black tiles	1	4	9	16	
Number of white tiles	2	6	12	20	$n^2 + n$
Total number of tiles	6	15	28	45	

4.2 Will any stage require 21 patterned tiles? Substantiate your answer with calculations. (3)

4.3 If the dimensions of a tile are 0,5m x 0,5m, write down a formula for the area of the pattern at any given stage *n*. (2)

[10]

Question 5

Thuli is planning to go on a round-the-world trip in 5 years' time. GlobeTrotters Travel is advertising a round-the-world package deal for R42 310.

5.1 If the rate of inflation remains at 7% for the next 5 years, how much can Thuli expect to pay for her trip? (Answer correct to the nearest ten rand.) (4)

5.2 Thuli will pay for the trip by saving a fixed amount from her salary each month for the next five years, starting at the beginning of this month and ending one month before her trip (60 payments). If the interest rate is 9,5% per annum, compounded monthly, how much must Thuli save each month in order to be able to pay for the trip in 5 year's time? (6)

[10]

Grade 12 Project: Finance **Marks: 50**

When borrowing money from a bank to buy a car, clients are given several options. Some of these options are explained below:

Fixed rate option*: The interest rate remains the same for the duration of the loan.*

Linked rate option*: The interest rate is linked to the Reserve Bank's prime lending rate. If the prime lending rate goes up or down, the interest rate for the loan goes up or down too.*

Payment Holiday Option*: The client is able to choose one month in the year where no repayment is made. These repayments are spread over the remaining months of the loan agreement.*

Balloon payment/Residual value option*: The amount that the client borrows is reduced by a given percentage of the loan amount (usually between 30% and 40%). The amount by which the loan was reduced has to be paid as a single, lump sum payment at the end of the loan period. The remaining 60% - 70% of the purchase price is covered by loan taken either as a fixed rate or a linked rate loan, with or without the payment holiday option.*

Use the data sheet provided to answer the following questions:

1 A client purchases a car on 1 January 2007. If interest is calculated monthly and the loan is repaid over four years, work out the monthly repayments on the purchase price of the car for both the fixed rate and linked rate loan options. Calculate the total cost of the car for both options. (8)

2 After 1 year (i.e. after the 12^{th} payment), the interest rate for the linked rate option increases, as indicated on the data sheet. Work out the new monthly payments and the total amount paid for the car. Compare your answers with your answers in question 1 and comment on the differences between the fixed- and linked rate options. (9)

3 A client selects the fixed rate option with a balloon payment of 30%. Calculate the balloon payment that will have to be made at the end of the loan period, as well as the monthly repayments on the loan. (6)

4 If the client decides to invest the difference between the fixed option payments in question 1 and the fixed option payments in question 3 and the interest rate on savings is 2% p.a. less than the fixed rate option for borrowing, how long will it take the client to save enough money for the balloon payment? (All interest is calculated monthly.) What is the total amount that the client has paid for the car? (6)

5 The client has decided to trade the car in after four years, pay the balloon payment option and buy a new car for cash.

5.1 Use the information provided on the data sheet to calculate the book value of the car after fours years. (2)

5.2 If the rate of inflation remains constant for the four year period, use the inflation rate given on the data sheet to calculate the cost of an entry level new car after four years. Give your answer correct to the nearest R1 000. (2)

5.3 Calculate how much the client will have to invest in a sinking fund each month in order to be able to pay cash for the new car. Use an interest rate that is 2% below the fixed interest rate for borrowing that is given in the data sheet. (4)

6 The client chooses the payment holiday option and decides to skip the last repayment of each year, develop a method to work out the client's monthly repayments. Calculate the monthly repayments and the total paid for the car. (5)

7 Write a newspaper article advising consumers about what they should consider when borrowing money to buy a car and which options are the most sensible. (Remember that interest rates can go up or down.) You should use the results of your calculations in the above questions to illustrate the points you make in your article. (8)

DATA SHEET

Data provided for 1 January 2007:

Purchase Price of car	R 120 000.00
Repayment period	48 months
Payment frequency (without payment holiday option)	12 payments p.a.
Payment frequency (with payment holiday option)	11 payments p.a.
Interest rate (fixed rate option)	13.5% p.a. compounded monthly
Interest rate (linked rate option)	12.3% p.a. compounded monthly
Interest rate after 1 year (linked rate option)	14.5% p.a. compounded monthly
Rate of depreciation on new vehicles on reducing balance	20% p.a.
Inflation rate	7.8% p.a.

Grade 12 Mathematics Exam — Paper 1

Time: 3 hours — **Marks: 150**

Question 1

1.1 Solve for *x*:

1.1.1 $(x-4)(x-3)=2$ (4)

1.1.2 $3x^2+2x+6 = 10$ (correct to TWO decimal digits) (5)

1.1.3 $(3x-2)^2 > 3x$ (6)

1.2 Solve simultaneously for x and y in the following set of equations:

$4y+3x=50$ and $x^2+y^2=100$ (7)

[22]

Question 2

2.1 Thandiswa invested R150 000 for a period of six years. The accumulated amount of her savings at the end of this period was R320 470. Calculate the effective interest rate per annum if the interest was compounded monthly. (5)

2.2 Lester wants to buy a car that costs R140 000. He intends paying a cash deposit of 18% and using a bank loan to repay the balance over 5 years in equal monthly repayments. The bank has quoted an interest rate of 13% p.a. compounded monthly.

2.2.1 Calculate the monthly repayments. (8)

2.2.2 If after two years Lester received a promotion at work and wants to increase his monthly repayment to R3 500 per month, how long will it take him to settle the loan. (Assume that the interest rate of 13% p.a. compounded monthly still applies). (7)

[20]

QUESTION 3

3.1 Researchers investigating the growth of HIV positive people (in millions) in a certain country arrived at a model of the data reflected in the following table.
(We take 2004 as $t = 0$, 2005 as $t = 1$ and so on.)

Year	**2004**	**2005**	**2006**	**2007**
Time (t), in years	0	1	2	3
Number of HIV positive people, (H) (in millions)	2,7	3,9	5,1	6,3

3.1.1 Make a conjecture about the relationship between the number of HIV positive people and time. (2)

3.1.2 Use your conjecture to write down the equation of H as a function of t. (2)

3.1.3 Use your equation to predict the number of HIV positive people in 2020. (3)

3.2 Given the geometric series: $5.(3)^4 + 5.(3)^3 + 5.(3)^2 + \ldots\ldots$

3.2.1 Explain why the series converges. (2)

3.2.2 Calculate the sum to infinity of the series. (3)

3.2.3 Calculate the sum of the first 9 terms of the series, correct to **TWO** decimal places. (4)

3.2.4 Use your answers to QUESTION 3.2.2 and QUESTION 3.2.3 to determine

$\sum_{n=10}^{\infty} 5.(3)^{5-n}$ (correct to **TWO** decimal places) (2)

3.3 Consider the sequence: 6 ; 10 ; 16 ; 24 ; 34 ; …

3.3.1 If the sequence behaves consistently, determine the next two terms of the sequence. (2)

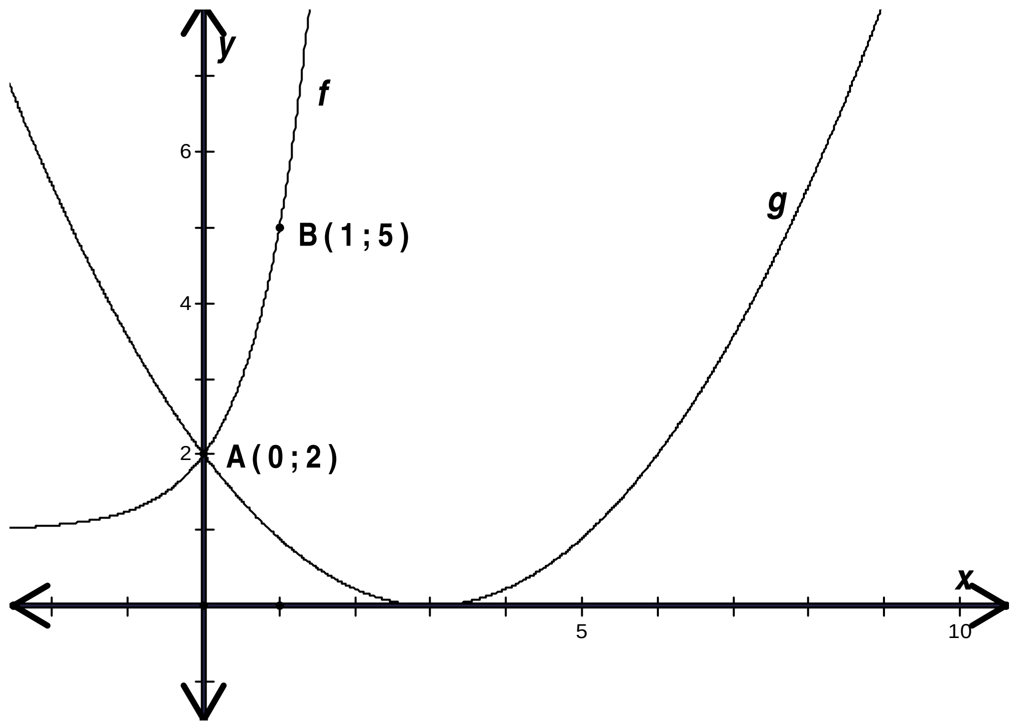
3.3.2 Calculate a formula for the n^{th} term of the sequence. (5)

3.3.3 Use your formula to calculate n if the n^{th} term in the sequence is 1264. (4)

[29]

Question 4

The sketch represents the graphs of the functions $f(x) = k^x + q$ and $g(x) = ax^2 + bx + c$.
The two graphs intersect at A(0 ; 2) and g touches the *x*-axis at $(3;0)$.
The coordinates of B, which lies on the graph of *f* are indicated.



4.1	Determine the value of *k* and *q*.	(3)
4.2	Determine the value of *a.*	(3)
4.3	Explain why the inverse of *g* is not a function.	(2)
4.4	Write down two ways in which the domain of *g* could be restricted in order that g^{-1} is a function.	(2)
4.5	Determine f^{-1}, the inverse of *f,* in the form *y* = ...	(2)
4.6	What is the defining equation of *h* if *h* is the reflection of *g* in the *y*-axis?	(2)

[14]

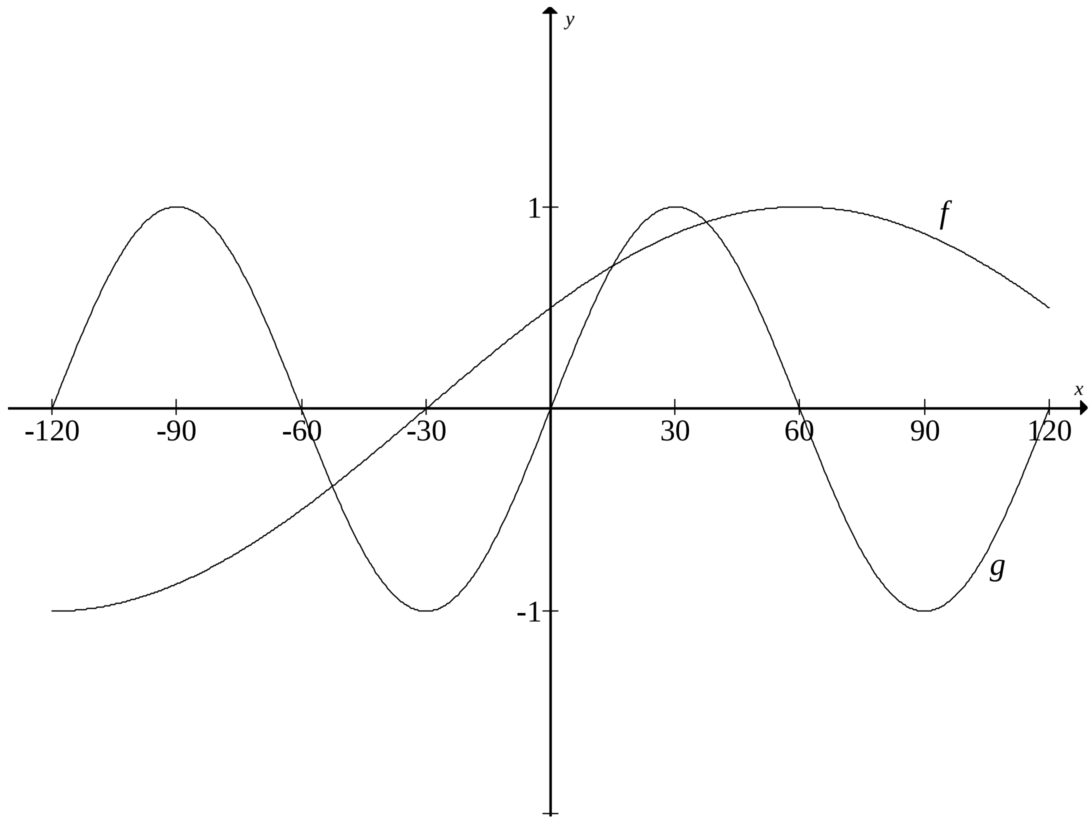
Question 5

5.1 Given: $g(x) = \dfrac{5}{x-2} - 4$

5.1.1 Write down the asymptotes of *g*. (2)

5.1.2 Calculate the intercepts of *g* with the axes. (3)

5.2 Sketched below are the functions $f(x) = \cos(x - 60°)$ and $g(x) = \sin 3x$ for $x \in [-180° ;\ 180°]$



5.2.1 Write down the period of *g*. (1)

5.2.2 Write down the new equation of *f* if it is shifted 30º horizontally to the right. (2)

5.2.3 Write down two values of x for which $f(x) - 1 = g(x)$. (2)

[10]

QUESTION 6

6.1 Calculate the derivative of the function $f(x) = -5x^2$ from **first principles**. (5)

6.2 Determine:

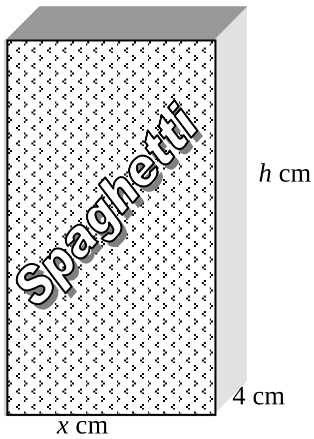
$$\frac{d}{dx}\left(4\sqrt{x} - \frac{8}{\sqrt{x}}\right)$$ (3)

6.3 The equation of a tangent to the curve of $f(x) = ax^3 + bx$ is $y - x - 4 = 0$. If the point of contact is (-1; 3), calculate the values of *a* and *b*. (7)

[15]

QUESTION 7

A pasta company has packaged their spaghetti in a box that has the shape of a rectangular prism as shown in the diagram below. The box has a volume of 540 cm³, a breadth of 4 cm and a length of *x* cm.



7.1 Express *h* in terms of *x*. (2)

7.2 Hence show that the total surface area of the box (in cm²) is given by:

$$A = 8x + 1080x^{-1} + 270$$ (3)

7.3 Determine the value of *x* for which the total surface area is a minimum. Round the answer off to the nearest cm. (4)

[9]

QUESTION 8

Given: $f(x) = 2x^3 - x^2 - 4x + 3$

8.1 Show that $(x-1)$ is a factor of $f(x)$. (2)

8.2 Hence factorise $f(x)$ completely. (2)

8.3 Determine the co-ordinates of the turning points of f. (4)

8.4 Draw a neat sketch graph of f indicating the co-ordinates of the turning points as well as the x-intercepts. (4)

8.5 For which value of x will f have a point of inflection? (4)

[16]

QUESTION 9

A small electronics company produces two models of television sets, the 'Plasma' and the 'LCD'. These televisions are sold to dealers at a profit of R3 000 per 'Plasma' and R4 000 per 'LCD'. A 'Plasma' requires 6 hours for assembly, 5 hours for finishing and checking. The 'LCD'requires 4 hours for assembly, 5 hours for finishing and checking . The total number of hours available per month is 240 in the assembly department and 250 in the finishing and checking department. The above information can be summarised by the following table:

Department	Hours for Plasma'	Hours for 'LCD'	Maximum hours available per month
Assembly	6	4	240
Finishing and checking	5	5	250

The company must produce at least 10 of each set.
Let x be the number of 'Plasma' and y be the number of 'LCD' models manufactured per month.

9.1 Write down the set of constraint inequalities. (4)

9.2 Use the graph paper provided to represent the constraint inequalities. (5)

9.3 Shade the feasible region on the graph paper. (2)

9.4 Write down the profit generated in terms of x and y. (2)

9.5 How many televisions of each model must be produced in order to maximise the monthly profit? (2)

[15]

Exam B

Grade 12 Mathematics Exam — Paper 1

Time: 3 hours — **Marks: 150**

Question 1

1.1 Solve for *x*:

1.1.1 $(x-4)^2 = 49$ (3)

1.1.2 $(x^2-2)(x+3) = x^3+6x$ (5)

1.2 Given $f(x) = (x+1)(x-5)$

1.2.1 Write down the *x* and *y* intercepts of *f*. (2)

1.2.2 Use the intercepts calculated in 1.2.1. to draw a rough sketch graph of *f*.
(It is not necessary to calculate turning points or to find axes of symmetry.) (2)

1.2.3 Use the sketch graph drawn in 1.2.2 (or any other method) to solve for *x* if:

$$(x+1)(x-5) \le 0$$ (1)

1.3 Solve for *x* and *y* in the following simultaneous equations:

$x^2 - x = 6 + y$ and $y = x - 3$ (5)

1.4 Solve for *x* and *y* if:

$(3x+2)^2 - (y-4)^2 = 0$ (2)

$(3x+2)^2(y-4)^2 = 0$ (1)

[21]

Question 2

2.1 Asisipho is a student who has a part-time job. She started on a salary of R1 500 per month and has received a 1,5% increase every two months.

2.1.1 If Asisipho has been working for one year, what is her salary? (3)

2.1.2 How much has Asisipho earned in total during the year that she has worked? (5)

2.2 If a motor car is damaged in an accident and the cost of repairs is more than 70% of the book value of the car, then the insurance company will write off the car. Shereen's car that she bought at the beginning of 2002 for R70 000, was damaged in an accident at the beginning of 2008. The estimated cost of repairs is R22 500.

2.2.1 Calculate the book value of the car, if its value has decreased 22% per year, calculated on diminishing balance. (4)

2.2.2 Determine whether or not the insurance company will write off the car. (3)

2.2.3 How much would Shereen have to have invested each month, from January 2002 until December 2008, at 7,5% interest per annum, calculated monthly, in order to have R80 000 to pay towards a new car? (4)

[19]

Question 3

3.1 Examine the following sequence of numbers:

1; 2; 4; 5; 7; 8; 10; 11; 13; 14; 16; 149

3.1.1 Give the next three numbers in the sequence. (3)

3.1.2 Describe the sequence in words. (2)

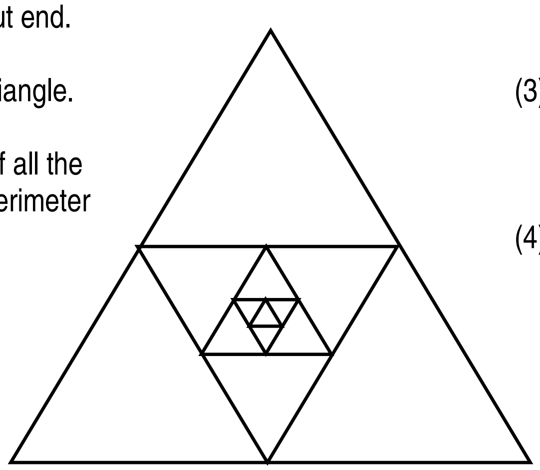
3.1.3 Calculate the number of terms in the sequence. (5)

3.2 If the n^{th} term of a series is given by $T_n = \frac{2n-1}{2^n}$, calculate $\sum_{k=1}^{4} \frac{2k-1}{2^k}$. (4)

3.3 In the diagram below, the first (outer) triangle is equilateral triangle with sides of 8 units. Another equilateral triangle in drawn within this triangle, by joining the midpoints of the sides of the outer triangle. This process is continued without end.

3.3.1 Calculate the perimeter of the fourth triangle. (3)

3.3.2 Show that the sum of the perimeters of all the inner triangles will never exceed the perimeter of the outer triangle. (4)



[21]

Question 4

Given: $f(x) = -2x^2$

4.1 Explain why, unless the domain of this function is restricted, its inverse is not a function. (2)

4.2 Write down the equation of inverse, $f^{-1}(x)$ of $f(x) = -2x^2$ for $x \in (-\infty;0]$ in the form $f^{-1}(x)=$... (3)

4.3 Write down the domain of $f^{-1}(x)$ (1)

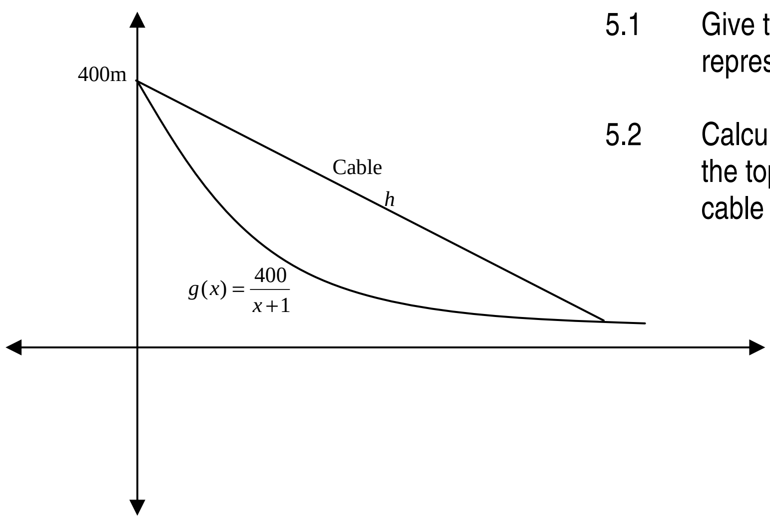
4.4 Draw graphs of both $f(x) = -2x^2$ for $x \in (-\infty;0]$ and $f^{-1}(x)$ on the same system of axes on the grid provided on the answer sheet. (4)

4.5 Show that $f^{-1}[f(-3)] = f[f^{-1}(-3)] = -3$ (4)

[14]

Question 5

A mountain cliff slopes according to the equation $g(x) = \frac{400}{x+1}$. A cable for a cable car is being erected from the top of the cliff. The gradient of the cable must be $-\frac{4}{5}$ and the cable is strung in a straight line, as shown below:



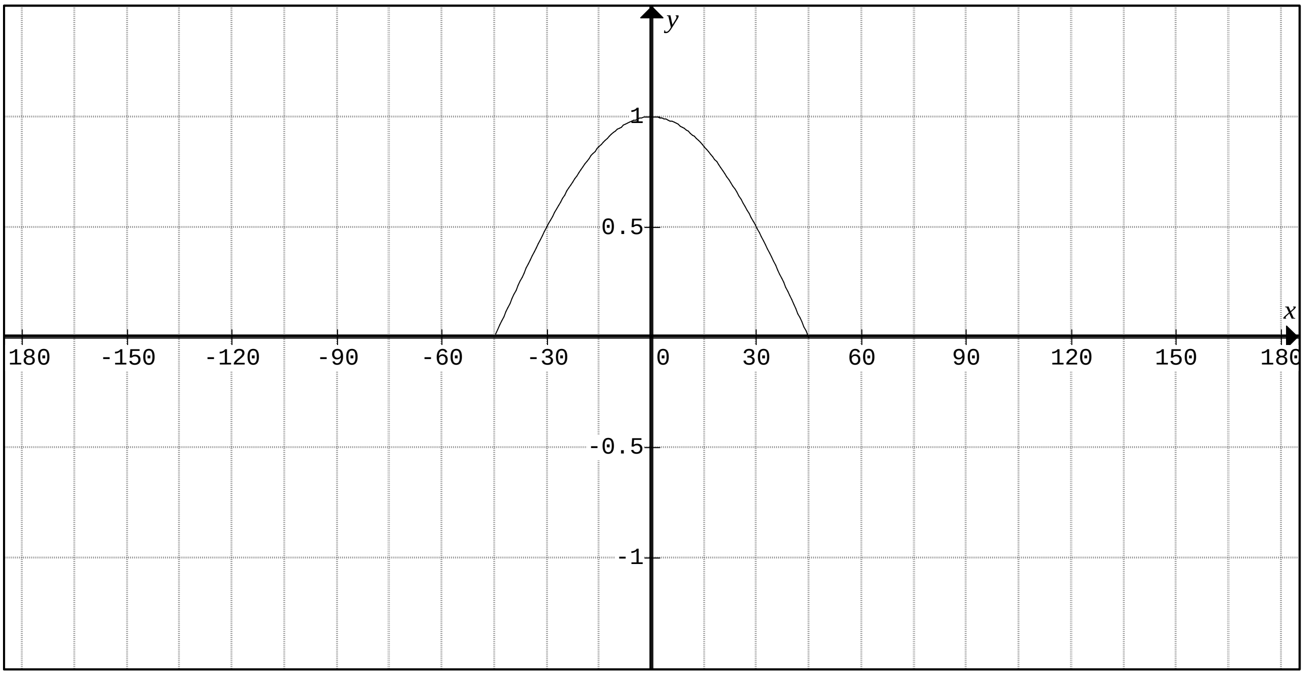
5.1 Give the equation of the straight line that represents the cable (*h*). (1)

5.2 Calculate the horizontal distance between the top of the cliff and the point where the cable touches the ground again (*P*). (5)

[6]

Question 6

A portion of the graph of the function $h(x) = \cos 2x$, $x \in [-180°;180°]$ has been sketched below.



6.1 Complete the graph on the diagram sheet provided. (1)

6.2 What is meant by the statement: "The period of the graph is $180°$? (2)

6.3 If the graph were shifted $30°$ to the left, how would this influence the values of the *x*-intercepts? (1)

6.4 What would the minimum value of *h* be if the graph were shifted vertically up by 1 unit? (1)

[5]

Question 7

7.1 Given $f(x) = -\dfrac{2}{x}$, determine $f'(x)$ from first principles. (5)

7.2 Determine the derivative of:

7.2.1 $f(x) = 7x^3 - 4x + 6$ (3)

7.2.2 $f(x) = 3\sqrt{x} - \dfrac{1}{3x}$ (4)

[12]

Question 8

Given $f(x) = x^3 + x^2 - 5x + 3$

8.1 Show that $(x-1)$ is a factor of $f(x)$. (2)

8.2 Factorise $f(x)$ fully. (3)

8.3 Determine the *x* and *y* intercepts of $f(x)$. (2)

8.4 Determine the co-ordinates of the turning point(s) of $f(x)$. (4)

8.5 Find the x-value of the point of inflection of $f(x)$ (1)

8.6 Draw a sketch graph of $f(x)$. (2)

8.7 For which value(s) of *x* is $f(x)$ increasing? (2)

8.8 Describe one transformation of $f(x)$ that, when applied, will result in $f(x)$ having two unequal positive real roots. (2)

8.9 Give the equation of *g* if *g* is the reflection of *f* in the *y*-axis. (3)

8.10 Determine the average rate of change of *f* between the points (0:3) and (1:0). (2)

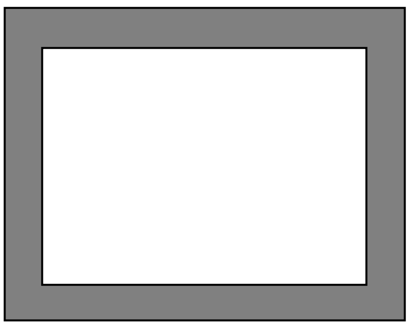
8.11 Determine the equation of the tangent to the *f* when *x* = -2. (4)

8.12 Prove that the tangent in 8.9 will intersect or touch the curve of *f* at two places. (4)

[31]

Question 9

A mirror is set into a wooden frame which is 2cm wide. The outside perimeter of the wooden frame is 72cm.



9.1 The length of the frame is *x* cm. Determine the breadth of the frame in terms of *x*. (1)

9.2 Determine the length and breadth of the mirror in terms of *x*. (2)

9.3 Show that the area of the mirror is given by the function: $A(x) = -x^2 + 36x - 128\ cm^2$ (2)

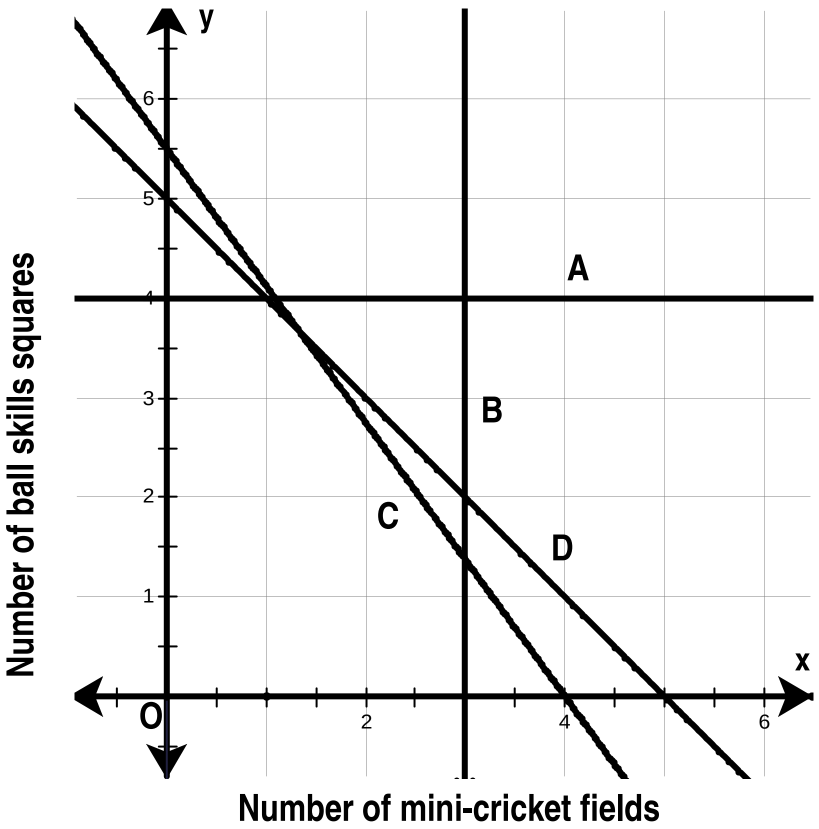
9.4 Calculate the dimensions of the mirror with the largest area that can fit into the frame. (4)

[9]

Question 10

A primary school is arranging it's sports programme. In order to ensure that all students are able to do sport, at least 55 students must do sport on an afternoon. A mini-cricket coach can take at most 14 learners at a time and a ball skills coach can take a maximum of 10 learners at one time. There are 5 coaches available, all of whom can coach either ball skills or mini-cricket. There are 3 mini-cricket fields and 4 ball skills squares available.

In the diagram below, *x* represents the number of mini-cricket fields used and *y* represents the number of balls skills squares used.



10.1 The constraints as outlined above have been graphed in the diagram and labelled A, B, C and D. Write down the letters A, B, C and D and then give the inequality that represents the constraint that has been graphed. (4)

10.2 Shade the feasible region. (1)

10.3 Which constraint is redundant? (1)

10.4 Write down all the feasible solutions to the problem. (2)

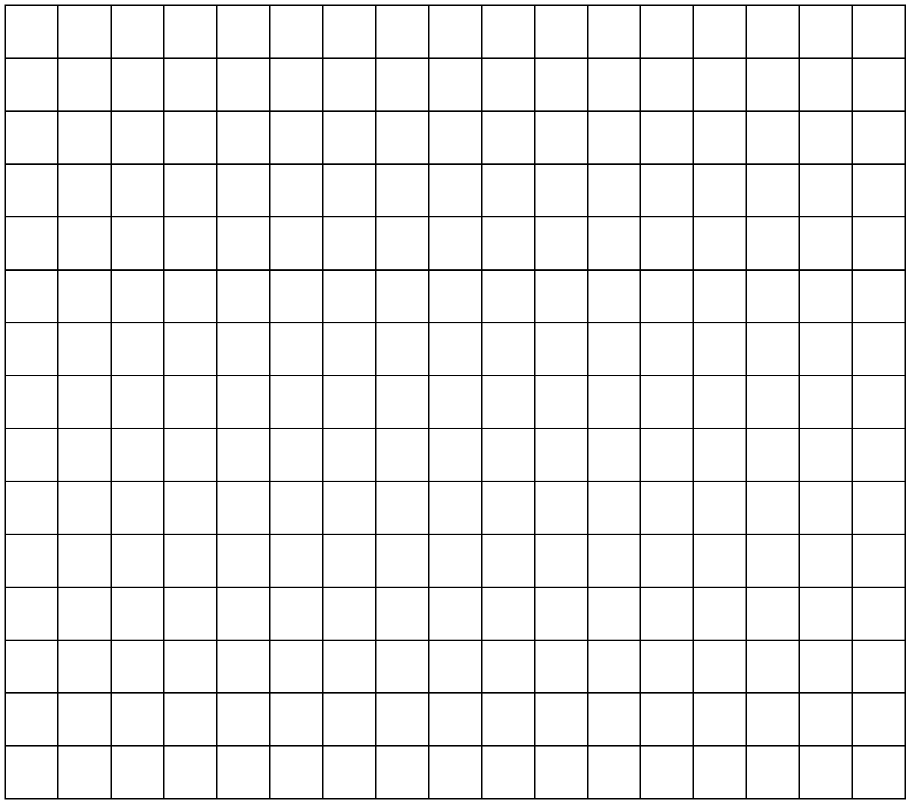
10.5 If all 5 coaches must be used, but it is cheaper to hire a ball skills square than a mini-cricket field, what is the most economical way to arrange sport on an afternoon? (1)

10.6 If it costs R12 per learner to do ball skills and R15 per learner to do mini-cricket, how much will the school pay the coaches in total for sports coaching on Mondays? (3)

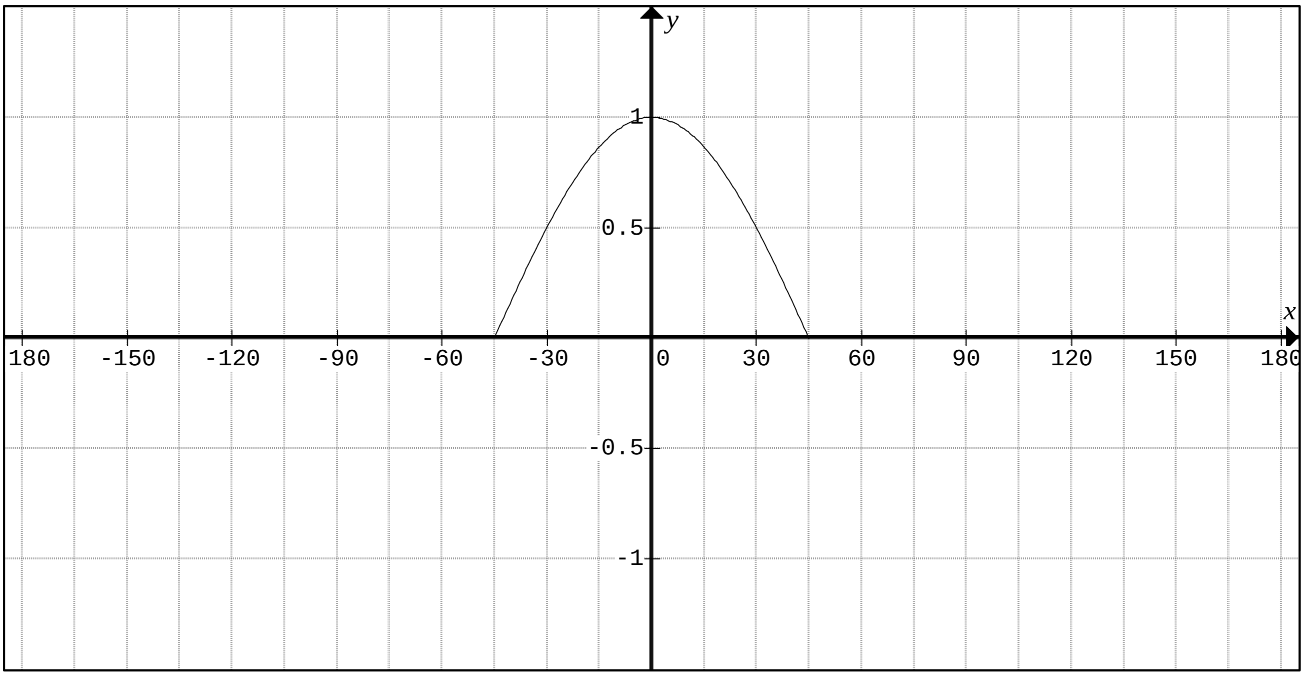
[12]

Diagram Sheet

Question 4



Question 6



Grade 12 Assignment: Recap of Grade 11 Data Handling — Marks: 50

From a matriculating class of 2007, 42 learners have been accepted at University and have raised the funds needed to follow their various choices.

Question 1:

The data below represents the fees being paid by each individual for their 1st year of study at university.

13,700	14,200	17,800	13,200	12,520	15,500
13,400	13,150	13,250	13,700	11,000	15,250
15,500	14,200	19,000	14,500	13,250	14,550
12,000	16,100	19,700	14,500	11,900	13,750
17,300	17,500	15,600	15,000	13,125	14,000
16,800	17,125	12,000	15,200	19,525	18,000
13,250	13,500	14,000	13,250	13,125	15,000

1.1 Using your calculator, determine the mean and standard deviation of this dataset (4)

1.2 Using the suggested classes shown on the diagram sheet, complete the columns for tally and frequency. (4)

1.3 Determine the median class. (1)

1.4 Draw an ogive for the given data on the grid provided. (6)

1.5 Using your ogive, draw a box-and-whisker plot to represent the data. (5)

1.6 Summarise your observations about the fees of different courses for 2008. (3)

[23]

Question 2:

Of the 42 learners going to university, 10 of them are following Science courses and needed Physical Science as one of their subjects. Their percentage for Physics in 2007 is as follows;

65	69	75	60	40
56	59	62	60	84

2.1 Determine, without using a calculator, the variance of percentage achieved in Physical Science by these learners. (4)

2.2 Find the Standard Deviation (2)

2.3 Describe, in words, what standard deviation tells us about a dataset. (2)

2.4 What can you deduce about these learners performance with respect to the standard deviation (3)

[11]

Question 3:

All learners wrote the Mathematics exam and their results are reflected in the back-to-back stem and leaf diagram below.

Boys				Stem	Girls					
			9	2						
		8	2	3						
3	2	0	0	4	0	2	7	7	8	
		3	2	5	2	2	7	9	9	9
			1	6	2	5	5	7	8	
	5	2	1	7	3	3	3	4		
6	5	4	1	8	8					
	2	2	1	9	2					

3.1 Determine both the mean and median of the boys and girls Mathematics marks (6)

3.2 Using these results and the shape of the diagram, describe the performance of these learners and compare the boys performance to that of the girls. (5)

[11]

Question 4:

The number of learners attending university from this school has grown over the last 7 years. Their records show the following;

Year of Matric	No. attending Tertiary Institution
2001	1
2002	2
2003	2
2004	5
2005	14
2006	25
2007	42

4.1 Draw a scatter plot to represent the data showing the growing numbers of learners going to University. (3)

4.2 Which function has the closest resemblance to the line of best fit for this graph? (2)

[5]

Assignment Diagram Sheet

Question 1

Class Interval	Tally	Frequency	Cumulative Frequency
less than 10 999			
11 000 - 11 499			
11 500 - 11 999			
12 000 - 12 499			
12 500 - 12 999			
13 000 - 13 499			
13 500 - 13 999			
14 000 - 14 499			
14 500 - 14 999			
15 000 - 15 499			
15 500 - 15 999			
16 000 - 16 499			
16 500 - 16 999			
17 000 - 17 499			
17 500 - 17 999			
18 000 - 18 499			
18 500 - 18 999			
19 000 - 19 499			
19 500 - 19 999			

Ogive showing University Fees of graduating Matric Class

Grade 12 Investigation: Polygons with 12 Matches **Marks: 100**

- How many different triangles can be made with a perimeter of 12 matches?
- Which of these triangles has the greatest area?
- What regular polygons can be made using all 12 matches?
- Investigate the areas of polygons with a perimeter of 12 matches in an effort to establish the maximum area that can be enclosed by the matches.
- Any extensions or generalisations that can be made, based on this task, will enhance your investigation. But you need to strive for quality rather than simply producing a large number of trivial observations.

Assessment

The focus of this task is on mathematical processes.
Some of these processes are: specialising, classifying, comparing, inferring, estimating, generalising, making conjectures, validating, proving and communicating mathematical ideas.

Marks will be awarded as follows:

40%	for communicating your ideas and discoveries, assuming the reader has not come across the task before. The appropriate use of diagrams and tables will enhance your communication.
35%	for the effective consideration of special cases.
20%	for generalising, making conjectures and proving or disproving these conjectures.
5%	for presentation: neatness and visual impact.

Grade 12 Test: Geometry and Trigonometry

Time: 1 hour **Marks: 50**

Question 1

1.1 Determine the equation of the circle, centre the origin, which passes through the point A $(-5;12)$. (2)

1.2 Show that the circle in 3.1 passes through the point B (13;0). (1)

1.3 Determine the equation of the line through A and B. (5)

1.4 Determine the equation of any tangent to the circle in 1.1 which is parallel to AB. Leave your answer in surd form. (6)

[14]

Question 2

Describe in words, each of the following transformations of the point $(a;b)$:

2.1 $(a;b) \rightarrow (-a;b)$ (2) 2.2 $(a;b) \rightarrow (a+1;-b)$ (2)

2.3 $(a;b) \rightarrow (a\cos\theta - b\sin\theta; b\cos\theta + a\sin\theta)$ (2)

2.4 $(a;b) \rightarrow \left(\frac{a-\sqrt{3}b}{2}; \frac{b+\sqrt{3}a}{2}\right)$ (2)

2.5 $(a;b) \rightarrow (kb; ka)$, where $-1 < k < 0$ (5)

[13]

Question 3

3.1 Determine the general solution of the equation $\sin 2x = \cos(x - 15^0)$ (6)

3.2 Mvuyo says 155^0 is a solution of the above equation. Is he right or wrong? Justify your answer. (2)

[8]

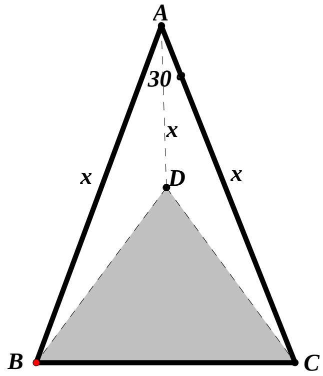
Question 4

Prove that $\tan(2x + 45^0) = \frac{2\tan x + 1 - \tan^2 x}{1 - \tan^2 x - 2\tan x}$

[6]

Question 5

5.1 Calculate, in terms of *x*, the length of the third side of the isosceles ΔABC with equal sides each *x* units and the angle between the equal sides 30^0. Leave your answer in surd form



(4)

5.2 Three identical triangles ΔABC, ΔABD and ΔADC are used to make a pyramid with equilateral triangle DBC as a base. Calculate the total surface area of the pyramid. Leave your answer in surd form. (5)

[9]

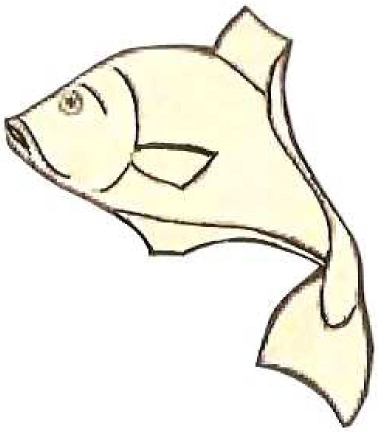
Grade 12 Project: Escher and Transformation Geometry — Marks: 100

Maurits Cornelis Escher, who was born in Leeuwarden, Holland in 1898, created unique and fascinating works of art that explore and exhibit a wide range of mathematical ideas. During his lifetime, he made 448 lithographs, woodcuts and wood engravings and over 2000 drawings and sketches.

Task 1

Escher became fascinated by the regular Division of the Plane in 1922. This is where an entire space is taken up with recurring images that tessellate with each other.

Using the outline of the fish shown below you need to attempt to 'divide the plane' with this shape.



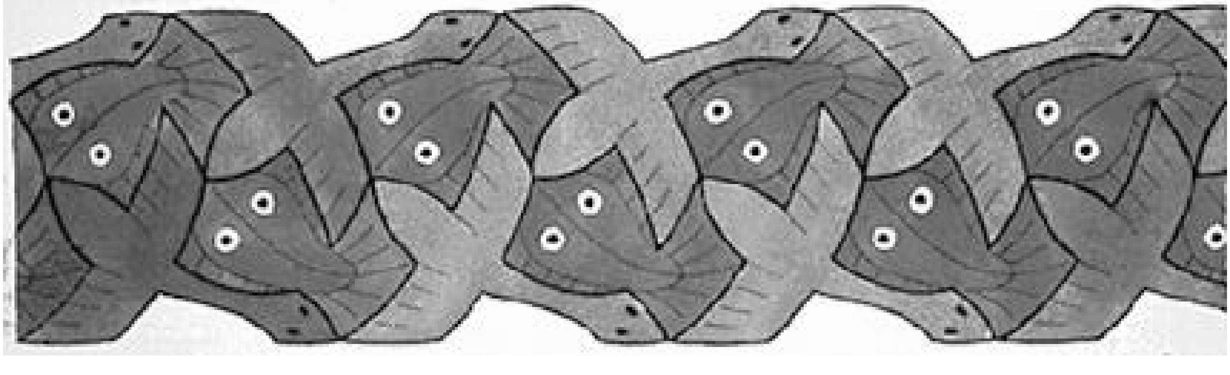
- Start with a blank sheet of A4 paper and choose a starting position to trace your fish and mark it clearly as the start.
- Using this shape alone, you must develop a pattern using the rules of transformation geometry. Each time you trace the fish in a new position, you need to describe the transformation you have carried out.
- You may use translation, reflection and rotation. In your description, remember to specify the degrees and direction that you have rotated the shape, or the manner in which you have reflected or translated it.
- The use of different colours often helps to keep track of where you are.
- Once you have finished your design with accompanying directions, a second person should be able to follow your instructions and come up with the same design.

Task 2

In the sketch below, Escher has used several different shapes to divide the plane.

- Identify how many shapes are used and what they are.
- Describe how they are combined and transformed to create the design shown.
- Make mention of any symmetry that exists, any reflection, rotation or translation that has taken place.

Task 3



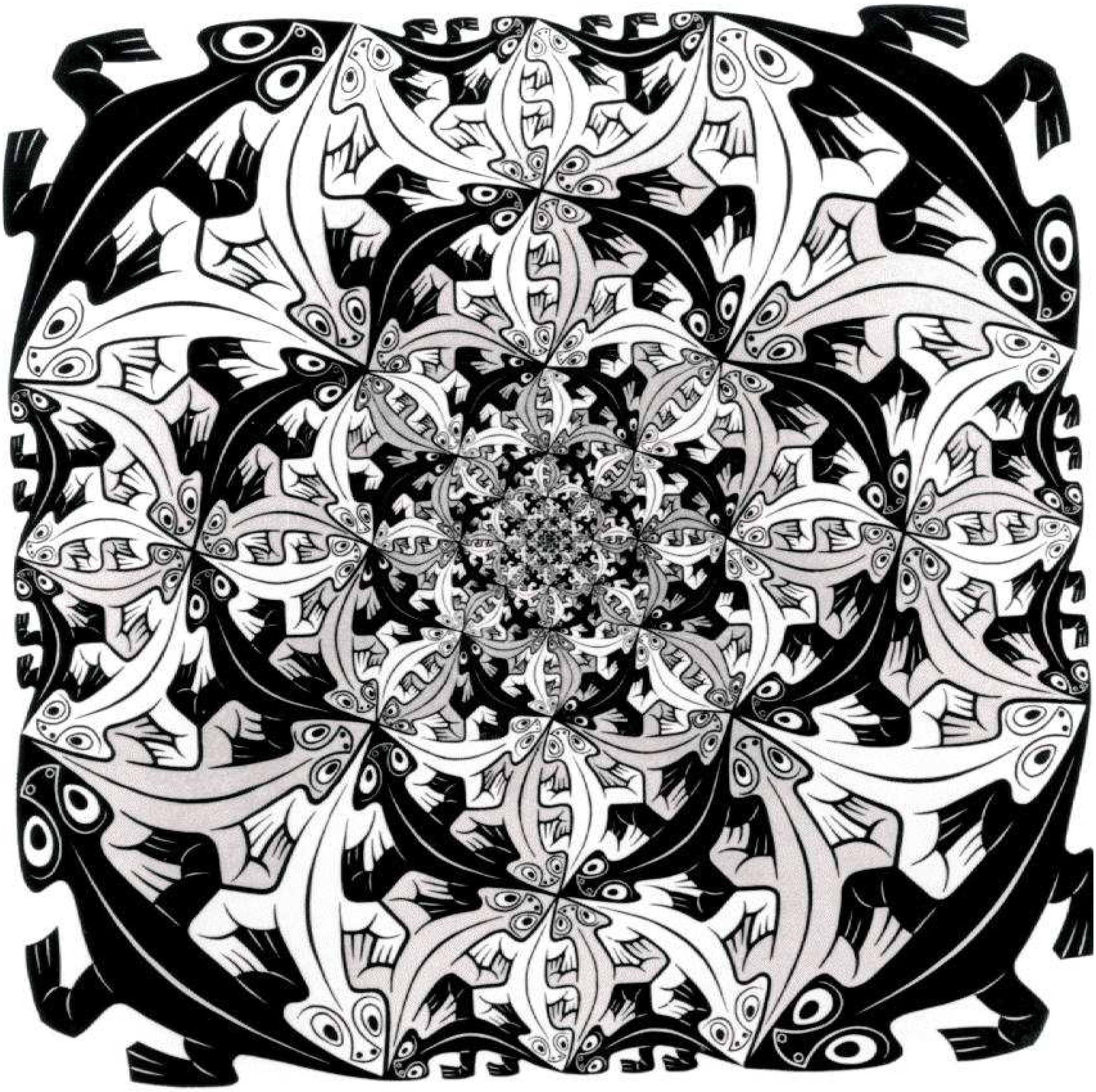
The above sketch shows a frieze where 2 shapes are tessellating to create a recurring pattern that can be used as a page border.

- Describe the transformation used in detail.

Task 4

Escher was mesmerized by geckos and did several sketches using this creature. Below is one of his more famous ones. This sketch uses a complex combination of transformations including enlargement, rotation and translation.

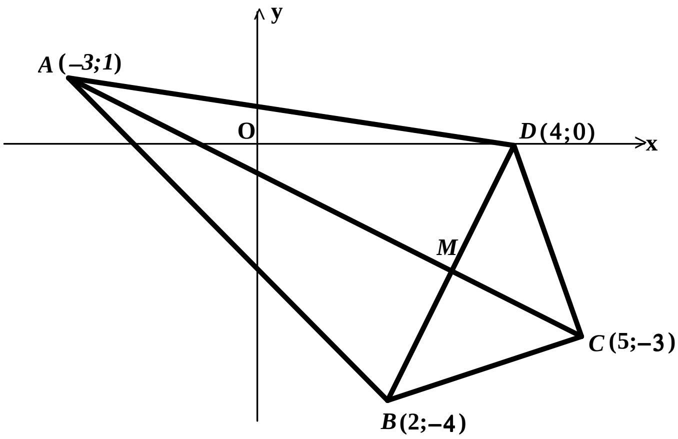
.



- Identify your own starting point and discuss as many different transformations as you can see.
- Be specific about ratios of enlargement and degrees and direction of rotation.
- There are several different ways of moving from one gecko to another, or from one group of geckos to another. Try to find as many patterns as you can.

Grade 12 Mathematics Exam **Paper 2**
Time: 3 hours **Marks: 150**

Question 1



1.1 Calculate the length of AC in simplest surd form. (2)

1.2 Calculate the equation of AC. (3)

1.3 Show that AC is the perpendicular bisector of DB. The equation of DB is $y = 2x - 8$ (8)

1.4 Determine the area of the kite ABCD (4)

1.5 Calculate the inclination of AB (2)

1.6 Hence or otherwise, calculate the size of $B\hat{A}D$, correct to the nearest degree. (4)

[23]

Question 2

2.1 Calculate the equation of the circle with centre M (2;–3) which passes through the point P(6;–1). (5)

2.2 Show that if Q is the point Q($0;-7$) the perpendicular bisector of PQ passes through the centre of the circle. (6)

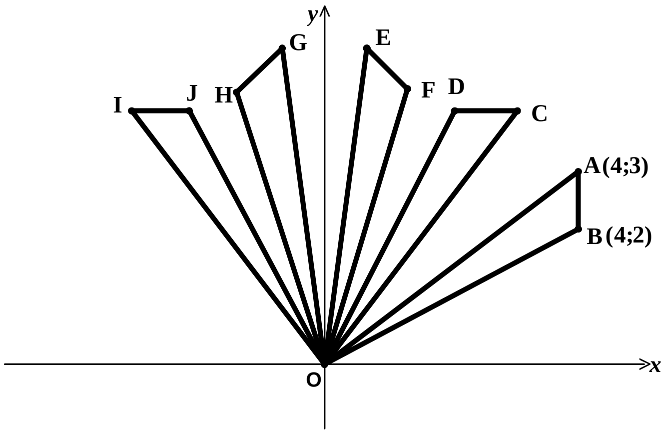
2.3 Does the point $R(-1;2)$ lie on the circle, inside the circle or outside the circle? Justify your answer. (3)

2.4 Calculate the images of M' of M and P' of P when the circle is enlarged, through the origin, by a factor of 1,5. (2)

2.5 Determine the ratio of the area of the original circle, centre M, passing through P to the area of the circle centre M', passing through P'. (4)

[20]

Question 3



In the given sketch, ΔOAB has been transformed by reflection and/or rotation, to create $\Delta OCD, \Delta OEF, \Delta OGH$ and ΔOIJ. In each case the co-ordinates of one vertex of the transformed triangle are given.

- Write down the co-ordinates of the other vertex that is not the origin and
- describe the transformation in words.

3.1 C is the point $(3;4)$ (4)

3.2 E is the point $\left(\frac{\sqrt{2}}{2};\frac{7\sqrt{2}}{2}\right)$ (4)

3.3 G is the point $\left(-\frac{\sqrt{2}}{2};\frac{7\sqrt{2}}{2}\right)$ (4)

3.4 I is the point $(-3;4)$ (4)

[20]

Question 4

4.1 Simplify, without the use of a calculator:
$\sin(90^0 + x).\cos(-x) - \tan(180^0 - x).\cos(180^0 + x).\sin(-x - 720^0)$ (5)

4.2 Given that $\sin\alpha = -\frac{3}{5}$ and $\cos\beta = -\frac{8}{17}$ where $\alpha, \beta \in [-90^0;180^0]$, calculate, without the use of a calculator, the value of:

4.2.1 $\tan(\alpha + \beta)$ (6)

4.2.2 $\sin 2\alpha$ (2)

4.2.3 $\cos\frac{\beta}{2}$ (in surd form). (4)

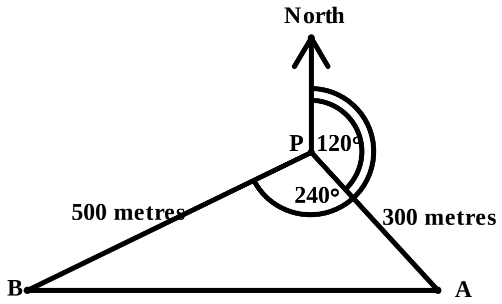
4.3 Determine the general solution of the equation: $1 + \sin 2\theta - 4\sin^2\theta = 0$,
correct to 1 decimal place where necessary. (8)

[25]

Question 5

The diagram below represents the course of a swimming race in a bay on the coast.

- P is the starting and finishing point;
- A is a buoy, 300 metres from P on a bearing of 120^0
- B is a buoy, 500 metres from P on a bearing of 240^0.



Calculate:

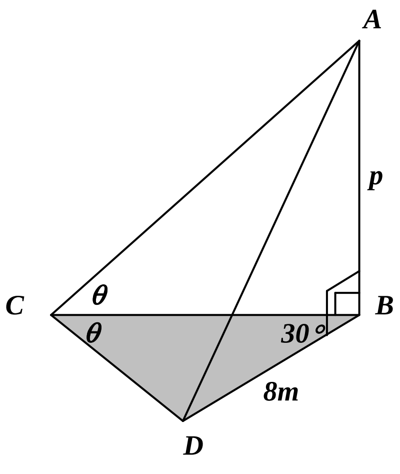
5.1 the bearing of the point P from the buoy, B and (1)

5.2 the distance competitors must swim (from P to A, then to B and then back to P). (6)

[7]

Question 6

In the sketch below, B, C and D are three points in the same horizontal plane. AB is a vertical pole *p* metres high. The angle of elevation of A from C is θ , $B\hat{C}D = \theta$, $C\hat{B}D = 30^0$ and $BD = 8$ metres.



6.1 Express $C\hat{D}B$ in terms of θ. (1)

6.2 Express BC in therms of p and a trigonometric ratio of θ . (2)

6.3 Hence or otherwise, show that $p = 4\left(1 + \sqrt{3}\tan\theta\right)$ (8)

[11]

Question 7

7.1 Solve the equation $\cos 2x = \sin(x + 30^0)$ for $x \in [-180^0; 180^0]$ (6)

7.2 Sketch graphs of $f(x) = \cos 2x$ and $g(x) = \sin(x + 30^0)$ on the same system of axes for $x \in [-180^0; 180^0]$. Show the co-ordinates of all points of intersection with the axes, all turning points and all points at which $f(x) = g(x)$ (8)

7.3 Read from your graph at least one value of x for which $g(x) - f(x) = 1{,}5$ (1)

[15]

Question 8

The masses, correct to the nearest kg, of ten girls and fifteen boys in a Grade 2 class are as follows:

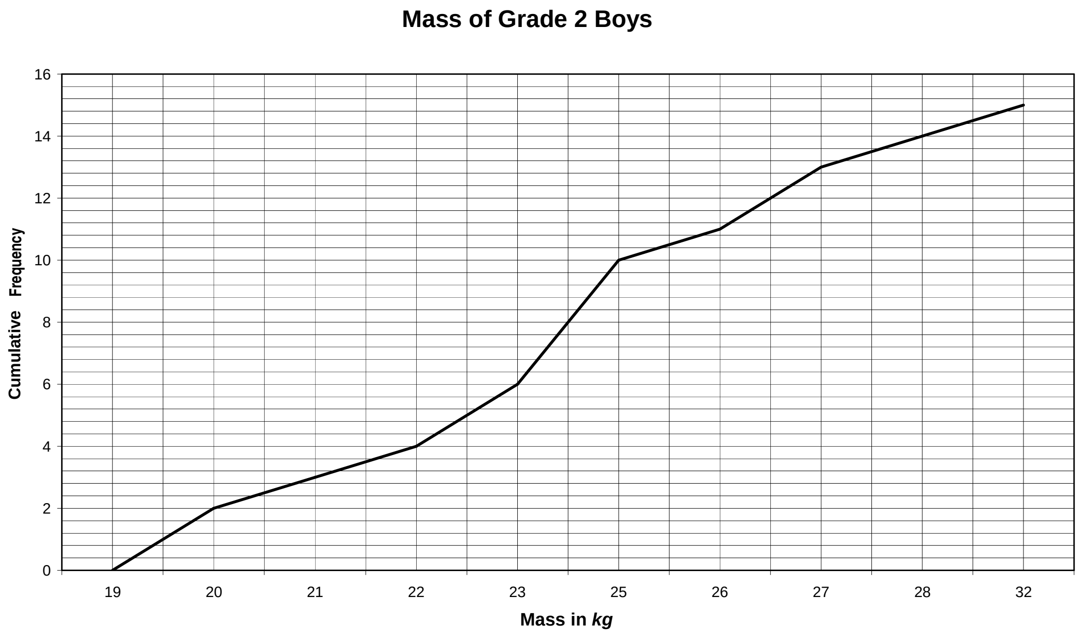
Girls: 19, 20, 23, 17, 30, 21, 20, 20, 21 and 22.
Boys: 26, 25, 25, 22, 28, 25, 27, 20,27,25,23, 32, 23, 21 and 20.

8.1 Draw a back to back stem and leaf plot to illustrate this data. (6)

8.2 Draw a box and whisker plot to illustrate distribution of the mass of the girls. (4)

8.3 Calculate the standard deviation of the masses of the girls; (4)

8.4 Your stem and leaf plot or the ogive given below may be useful in answering the following questions:



8.4.1 What is the modal mass of the boys (1)

8.4.2 Write down the inter-quartile range of the masses of the boys (4)

8.4.3 the approximate value of m, judged from the given sample, if ninety percent of grade 2 boys have a mass of less than $m\ kg$. (2)

[21]

Question 9

When data is normally distributed about the mean, 68% of the data is within one standard deviation of the mean. Decide whether the following statements are true or false and explain your answer:

9.1 There are more data items within one standard deviation of the mean than in the inter-quartile range of normally distributed data. (2)

9.2 When the mean is less than the median, the data is said to be negatively skewed. (2)

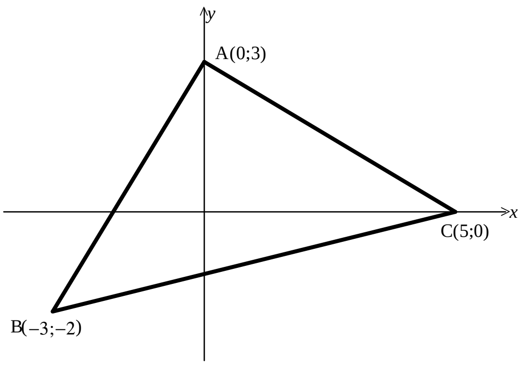
9.3 A cylindrical dam is emptying at a rate of $x\ litres$ per hour and the height of the water is recorded at regular time intervals. A scatter plot of height against time will show a positive correlation. (2)

9.4 The shape for the scatter plot described in 9.3 will be a parabola. (2)

[8]

Grade 12 Mathematics Exam **Paper 2**
Time: 3 hours **Marks: 150**

Question 1



1.1 Show that ΔABC is a right angled isosceles triangle. (6)

1.2 Determine the area of ΔABC (3)

1.3 Given that BC is a diameter of the circumscribed circle of ΔABC, show that the centre of this circle is M, the point $(1;-1)$ (2)

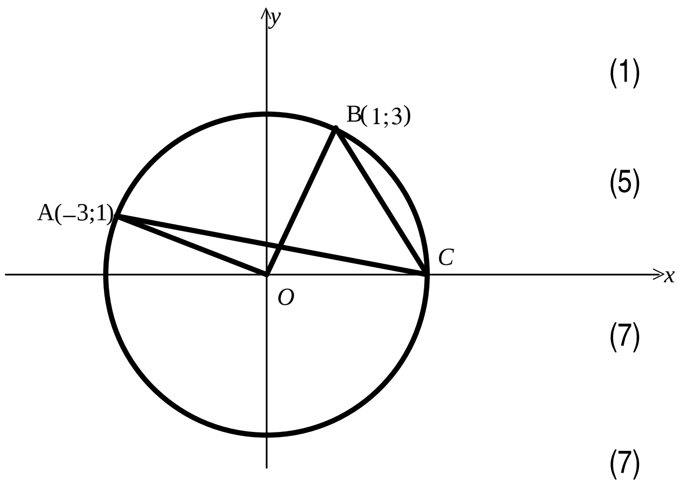
1.4 Calculate the equation of the circumscribed circle of ΔABC (4)

1.5 Determine the equation of the tangent to the circle at C and show that this tangent is parallel to MA. (8)

[23]

Question 2

A and B are points on the circle $x^2 + y^2 = 10$. C is the point where the circle cuts the positive x-axis.



2.1 Write down the radius of the circle. (1)

2.2 Determine the equation of the straight line AB . (5)

2.3 Determine the co-ordinates of the points at which the line $y = -2x$ cuts the circle. (7)

2.4 Given that $A\hat{O}B = 90^0$, show that $A\hat{O}B = 2 \times A\hat{C}B$ (7)

[20]

Question 3

3.1 P is the point $(-1;5)$, write down the image of P after each of the following transformations:

3.1.1 Reflection about the line $y = x$ (2)

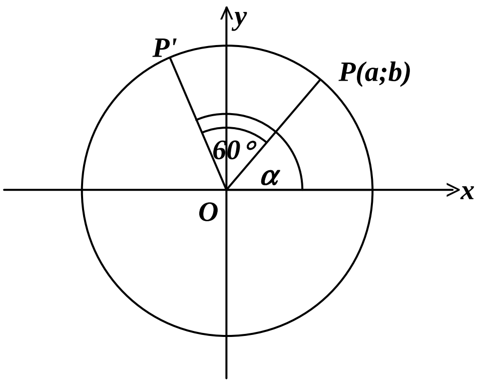
3.1.2 Rotation about the origin through an angle of 180^0 (2)

3.2 Describe in words the effect of the given transformation on the vertices and on the area of any ΔABC:

3.2.1 $(x;y) \to \left(\frac{x}{2};\frac{y}{2}\right)$ (4)

3.2.2 $(x;y) \to (x+2;-y)$ (6)

3.3 Show, in detail, that the image of any point P$(a;b)$, after rotation through an angle of 60^0 about the origin is $\left(\frac{a}{2}-\frac{b}{2};\frac{b}{2}+\frac{\sqrt{3}a}{2}\right)$



(6)

[20]

Question 4

4.1 Simplify: $\dfrac{\sin(-\alpha).\cos(90^0-\alpha)}{\cos\alpha.\cos(180^0+\alpha)}$ (5)

4.2 Given that $\sin 27^0 = t$, express each of the following in terms of t:

4.2.1 $\cos 27^0$ 4.2.2 $\tan 153^0$ (2, 3)

4.2.3 $\cos 243^0$ 4.2.4 $\cos 54^0$ (2, 3)

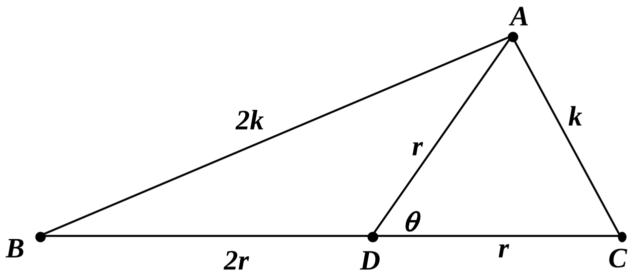
4.3 Determine the general solution of the equation: $\tan(3x+75^0)+1=0$ (5)

4.4 Simplify, without using a calculator: $\dfrac{\sin 15^0}{2}+\dfrac{\sqrt{3}\cos 195^0}{2}$ (5)

[25]

Question 5

In ΔABC, $A\hat{D}C = \theta$, $DA = DC = r$, $BD = 2r$, $AC = k$ and $BA = 2k$



5.1 In ΔADC, express $\cos\theta$ in terms of r and k (2)

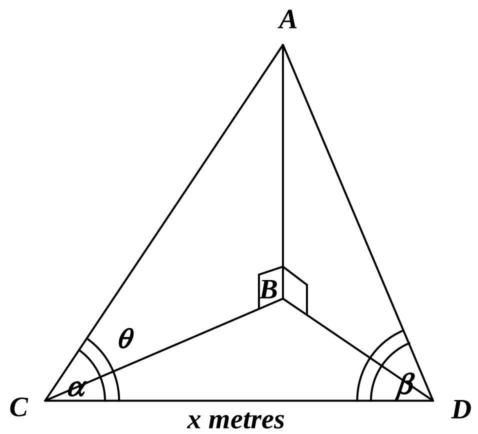
5.2 In ΔABD, express $\cos\theta$ in terms of r and k (3)

5.3 Hence show that $\cos\theta = \frac{1}{4}$ (3)

[8]

Question 6

AB .is a vertical tower in a horizontal plane BCD. The angle of elevation of A from C is θ, $A\hat{C}D = \alpha$, $A\hat{D}C = \beta$ and the distance $CD = x$ *metres*.



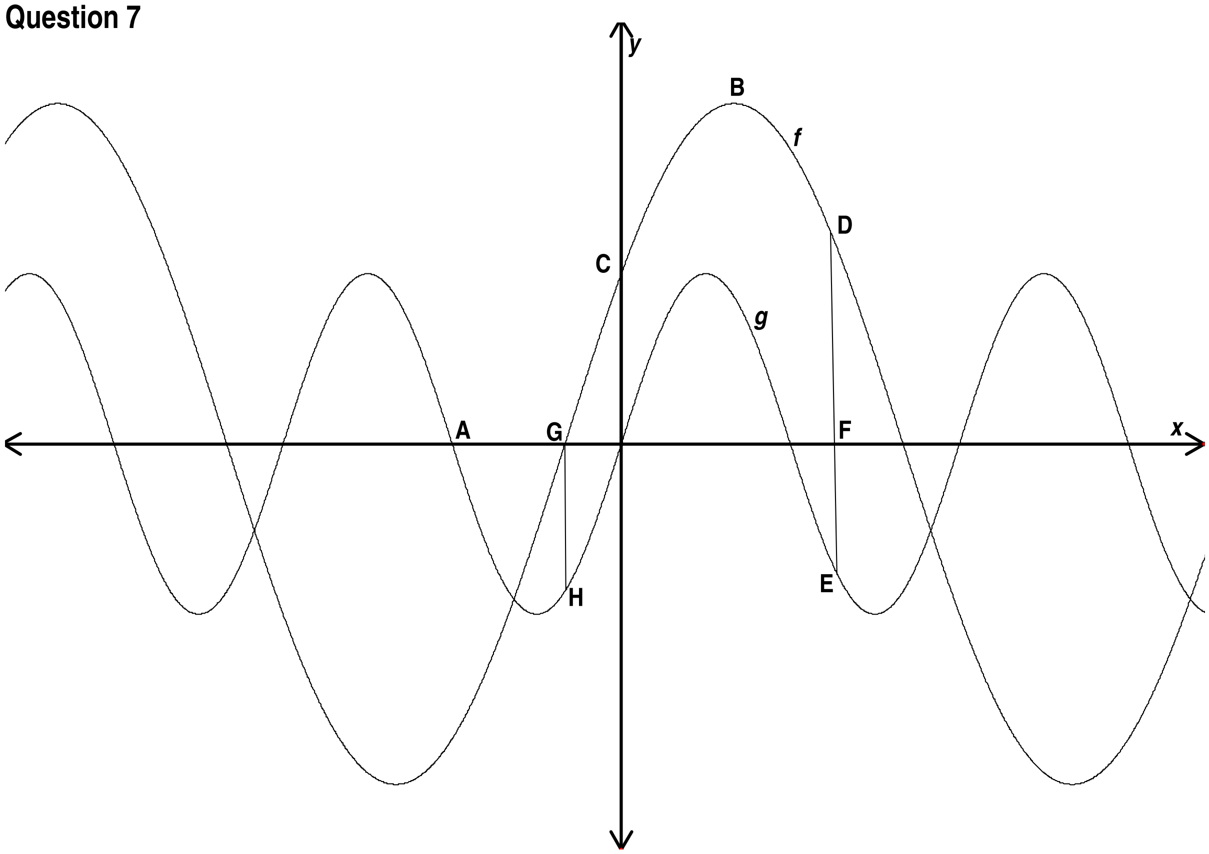
6.1 Prove that the height of the tower AB = $\dfrac{x\sin\theta\sin\beta}{\sin(\alpha+\beta)}$ (5)

6.2 Calculate the height of the tower if x = 40 *metres*, $\alpha = 50^0$, $\beta = 70^0$ and $\theta = 15^0$. (2)

6.3 Calculate the area of ΔACD (3)

[10]

Question 7



Sketched above are graphs of $y = f(x) = 2\cos(x - 60^0)$ and $y = g(x) = \sin 2x$. A is an intercept with the x axis, B is a turning point and C is an intercept with the y axis.

7.1 Calculate the co-ordinates of A, B and C (5)

7.2 Calculate the distance DE if F is the point $(120^0;0)$, $DFE\|$ the y axis, with D and E on the graphs as shown. (5)

7.3 Write down the equation of the graph through G if it were shifted down so that G co-incided with H. G and H lie on the graphs as shown and $GH \perp$ the x-axis. (5)

[15]

Question 8

The masses, correct to the nearest 10 g of eight tomatoes are: 190, 160, 150, 230, 220, 180, 180 and 170.

8.1 Use the formula $\sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n}$ for variance to calculate the standard deviation of the eight tomatoes. (4)

8.2 Explain what the standard deviation tells us about a set of data. (2)

[6]

Question 9

The following data was collected and recorded as shown in the table

Marks	Frequency	Cumulative frequency
0 to 29	2	2
30 to39	10	12
40 to 49	43	55
50 to 59	72	127
60 to 69	53	180
70 to 89	37	217
80 to 99	25	242
90 to 100	3	245

9.1 Draw an ogive to illustrate the data in the table. (8)

9.2 Read from your graph the median and lower and upper quartiles, showing where readings have been taken. (6)

9.3 Calculate the approximate mean from the data, showing how your answer was obtained. (6)

9.4 Compare the approximate mean, median and mode and hence comment on the nature of the distribution of the data. (3)

[23]